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Risk Acceptance Criteria and/or Decision Optimization

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Summary

Acceptance criteria applied in practical risk analysis are recapitulated including the concept of risk profile. Modelling of risk profiles is illustrated on the basis of compound Poisson process models. The current practice of authoritative acceptance criteria formulation is discussed from a decision theoretical point of view. It is argued that the phenomenon of risk aversion rather than being of concern to the authority should be of concern to the owner. Finally it is discussed whether there is an ethical problem when formally capitalising human lives with a positive interest rate.

1. Introduction

Any society of humans has formulated regulating rules for the activity of its members. In particular this is the case for risky activities that may accidentally cause loss of life, limb and property not only for the active persons themselves. The purpose of the regulations is to eliminate the risks or at least to make the risks marginal as compared to the benefits of the activities.

For larger projects or operations the regulations often contain so-called acceptance criteria through which the authorities put restrictions on the engineering decisions and the operation practice with the intention of keeping the occurrence rate of different categories of adverse events below some specified limits. Such restrictions amount to a loss of benefit both to the society and to the individual owners. It is therefore desirable if some rational optimization principles could be established by use of which the authority could support its decisions about the quantitative formulation of the acceptance criteria. The past practice seems not to indicate that such optimization principles have been taken seriously into consideration. Rather present practice of formulating acceptance criteria seems to be based on the acceptance of the actually observed occurrence rates of the different categories of adverse events.

This paper recapitulates the acceptance criteria formalisms as they seem to appear most frequently in present practice of risk analysis. A definition is given of a practicable and conceptually simple mathematical modelling tool suited for accounting the benefit loss accumulation due to the random occurrence of adverse events. The elementary building stone for this model is the compound Poisson process, for which there is an asymptotic expression for the probability distribution of the first passage of the accumulated loss through a given level of loss. This asymptotic expression (as level $\to \infty$) is advocated to be used after calibration to the correct mean and standard deviation as an estimate of the so-called risk profile of the considered project (ship, structure, operation) to be compared with the acceptance criteria given by the authorities.

After the introduction of these mathematical tools the risk acceptance problem is discussed in the light of rational decision theoretical concepts. The decision problem of the authority is discussed and it is argued that the risk aversion phenomenon should be of concern to the owner rather than to the authority in spite of what presently seems to be accepted practice. Finally it is discussed whether it is an ethical problem to capitalise human life by use of a positive interest rate.

2. Some current risk acceptance criteria judged by related expected cost of failure

Risk acceptance criteria are often given in the form of bounds on the annual probability of failure in dependence of the consequence of failure measured as monetary costs or lives lost. One bound specifies the accepted level of the failure probability and another larger bound specifies a level which is just marginally accepted. Both bounds decrease with the consequence of failure. The interval between the two bounds are considered as a grey zone where more detailed analysis and argumentation may be required in order to reduce the uncertainty of the assessed probability of failure so that acceptance may possibly be obtained from the superior authorities. An acronym for the grey zone is the ALARP zone (As Low As Reasonably Practicable) indicating that if the benefits of the device (ship, structure, etc.) or the operation are desired and the costs of risk reduction are not disproportionate to the benefits, then such reduction should be undertaken and the proper authority may declare the residual risk as being tolerable, the obtained benefit considered. Examples of such bounds together with indications of domains of experienced fatalities are referred in Bea (1990).

It seems reasonable to assume that the decrease of the bounds with the failure probability should be based on a decision theoretical principle stating that the expected cost of failure should be independent of the probability of failure. To see the implication of this principle let $\mathbf{1}(F)$ be the random indicator of failure within a given year of operation of the considered device, i.e. $\mathbf{1}(F)=1 \;\; if \;\; failure\ occurs$, $\mathbf{1}(F)=0 \;\; otherwise$, and let C be the random cost triggered by a failure of the device. Then the expected cost of failure is

$$E[C\mathbf{1}(F)] = E\Big[E[C|\mathbf{1}(F)]\mathbf{1}(F)\Big] = cp \qquad (2.1)$$

where $c = E[C]$ is the expected cost and $p = E[\mathbf{1}(F)]$ is the failure probability. If the expected cost of failure is required to be less or equal to a constant k, it follows that the equation of the acceptability bound becomes

$$c = \frac{k}{p}. \tag{2.2}$$

The acceptance bound referred in Bea (1990) has the equation

$$c^a = \frac{k}{p} \tag{2.3}$$

with $a \cong 0.7$, that is, the bound on the expected cost cp of failure increases with the expected cost c triggered by the failure. This finding is surprising because a frequently stated argument for setting risk acceptance criteria is that judged by the public reactions it is worse to have a big loss by a single accident than the same loss distributed in time over several accidents. This risk aversion attitude should rather impose a risk acceptance criterion with the property that the bound on the expected cost cp decreases with the mean consequence.

It seems as if the acceptance bounds referred in Bea (1990) are simply formulated on the basis of statistics on experienced failures. Such an acceptance bound serves as a warning about risks that may deviate from normal experience, but it does not serve as an instrument for improving the safety relative to the experience. In fact, an elementary mathematical consideration explains why the experienced frequency of failures of a given large consequence size is the smaller the larger the consequence size, Ditlevsen (1995).

3. Risk profile acceptance bounds

Acceptance criteria may be specified as above with bounds for acceptable probabilities of single adverse events with given consequences. In practice another not completely equivalent formulation of the acceptance criteria is often used for specific devices or operations. A total risk account is set up and a corresponding risk profile is calculated. The T year risk profile is defined as the complementary distribution function

$$\overline{F}_{L(T)}(x) = 1 - F_{L(T)}(x) = P[L(T) > x] \tag{3.1}$$

of the total accumulated loss $L(T)$ resulting from all relevant adverse events occurring during T years. In the following L will be used as a short notation for $L(T)$.There seems to be no standard value for T, but usually the reference period for the acceptance criterion is $T = 1$ year (at least for casualties). Complete acceptance is then obtained if $\overline{F}_L(x) \le A(x)$ for all x, where $A(x)$ is some specified risk profile bound. Marginal acceptance is obtained if $\overline{F}_L(x) \le B(x)$ for all x with $\overline{F}_L(x) > A(x)$ for some x, where $B(x) > A(x)$ is some specified upper risk profile bound. The domain between $A(x)$ and $B(x)$ is the ALARP domain. If $\overline{F}_L(x) > B(x)$ for some x, then the risk profile is declared non-acceptable.

It is not unusual to see definitions of $A(x)$ and $B(x)$ that correspond to decreasing straight lines in diagrams with logarithmic scale on both axes, that is, $A(x) = k/x^a$ for some $a, k > 0$, and similarly for $B(x)$. Thus $A(x) = \min\{1, kx^{-a}\}$. If $\bar{F}_L(x) \le A(x)$ for all x, the expected total loss becomes bounded by

$$E[L] = \int_0^\infty \bar{F}_L(x)dx \le \int_0^\infty A(x)dx = k^{1/a} + \int_{k^{1/a}}^\infty kx^{-a}dx = \infty \text{ for } a \le 1 \text{ and } ak^{1/a}/(a-1) \text{ for } a > 1 \quad (3.2)$$

If $A(x)$ is specified with some $a \le 1$ and if it should happen that $\bar{F}_L(x) \equiv A(x)$, it follows that the acceptance criterion is satisfied even though the expected loss is infinite. According to the decision theory of Von Neumann and Morgenstern (1943) a decision giving infinite expected loss cannot be optimal and the operation should not be undertaken. To avoid this conflict it is often used to cut off the bounds $A(x)$ and $B(x)$ at some large level x_0 of x such that $A(x) = B(x) = 0$ for $x > x_0$, or $A(x)$ and $B(x)$ are chosen such that the decrease for large x is at least as fast as kx^{-a} for some $a > 1$. However, it is not essential to require that the bounds $A(x)$ or $B(x)$ have finite integrals because any realistic modelling of the distribution function $F_L(x)$ will possess a finite expectation $E[L]$. The identity $\bar{F}_L(x) \equiv A(x)$ can simply not occur in practice if $A(x)$ has infinite integral.

4. Risk profile based on compound Poisson process modelling of accumulated loss for given adverse event category

The possible adverse events are identified by use of different risk analysis techniques such as hazard scenarios and event trees for the detection of causal sequences of happenings that can lead to adverse events. The sequence of adverse events as occurring during time is usually made up of subsequences belonging to different categories. For example, for ships some adverse event categories are fires, collisions between ships, impacts against fixed structures, groundings, capsizing, etc. Several of these categories can be divided further into subcategories. In the mathematical modelling an adverse event category may suitably be characterised by homogeneity (unimodality) of the probability distribution of the loss resulting from the adverse event. For each category the sequence of occurrence time points of the adverse events can be modelled in several ways dependent on the actions taken after each occurrence.

The conceptually most elementary and possibly also the most generally applicable model with respect to operability is the homogeneous Poisson point process $N(t)$ (= number of points up to time t). Given that the losses $X_1, X_2, \ldots$ of the sequence are mutually independent and identically distributed as a random variable X, and the losses occur at the time points $\tau_1, \tau_2, \ldots$, respectively, then the random process of the accumulated loss capitalized to time zero can be written as

$$L(t) = \sum_{n=1}^{N(t)} X_n e^{-\gamma \tau_n} \quad (4.1)$$

where γ is the interest rate. (The exponential form of the capitalization factor is a mathematically convenient and more than sufficiently accurate continuous simplification of the discrete interest addition used in bank business). The cumulative process $L(t)$ is an inhomogeneous compound Poisson process which is homogeneous only for $\gamma = 0$.

The calculation of the T-year risk profile $\bar{F}_L(x)$ then amounts to calculating the probability

$$\bar{F}_L(x) = P[L(T) > x] = \sum_{n=0}^{\infty} P\left[L(T) > x \middle| N(T) = n\right] p_n = \sum_{n=0}^{\infty} P\left[\sum_{i=1}^{n} X_i e^{-\gamma T U_i} > x\right] p_n \quad (4.2)$$

where $U_1,\ldots,U_n$ are mutually independent random variables distributed as a random variable U of uniform distribution on the interval from 0 to 1, and independent of $X_1, X_2,\ldots$. The weights $p_n = P[N(T) = n]$ in the sum are the probabilities in the Poisson distribution of mean λT where λ is the intensity of the Poisson point process. It follows directly from the form of the right side of (4.2) that $P[L(T) > x] = P\left[\tilde{L}(T) > x\right]$ with $\tilde{L}(T)$ being the homogeneous compound Poisson process

$$\tilde{L}(t) = \sum_{n=1}^{N(t)} \tilde{X}_n \quad (4.3)$$

where $\tilde{X}_1, \tilde{X}_2,\ldots$ are distributed as $\tilde{X} = X e^{-\gamma T U}$ with mean $E\left[\tilde{X}\right]$ and coefficient of variation $V_{\tilde{X}}$ determined by

$$E\left[\tilde{X}\right] = E[X] E\left[e^{-\gamma T U}\right] = E[X] \frac{1}{\gamma T}(1 - e^{-\gamma T}) \quad (4.4)$$

$$1 + V_{\tilde{X}}^2 = (1 + V_X^2) \frac{\gamma T}{2} \frac{1 + e^{-\gamma T}}{1 - e^{-\gamma T}} \quad (4.5)$$

respectively.

5. Asymptotic risk profiles

If the terms in (4.2) of smaller order than λ as $\lambda \to 0$ are neglected, we get asymptotically

$$\bar{F}_L(x) \approx \lambda T\, P(X e^{-\gamma T U} > x) \quad (5.1)$$

Another asymptotic expression is available in the limit $x / \mu \to \infty$ where μ is the mean of $\tilde{X}$. Since $\bar{F}_L(x) = P(\tau \le T)$, where τ is the time to first passage of a sample function of the process $\tilde{L}(t)$ through the level x, it follows from a result in Ditlevsen (1990) that

$$\bar{F}_L(x) \approx \int_0^{\infty} g(u) F(\lambda T; u)\, du \quad (5.2)$$

asymptotically as $x / \mu \to \infty$. The integrand in (5.2) is the product of the gamma distribution function $F(\lambda T; u) = \int_0^{\lambda T} t^{u-1/2} e^{-t} dt / \Gamma(u + 1/2)$ and the inverse Gaussian density $g(u) = \dfrac{x/\mu}{V u \sqrt{u}} \varphi\left(\dfrac{u - x/\mu}{V\sqrt{u}}\right)$, where $V = V_{\tilde{X}}$ and $\varphi(x) = \dfrac{1}{\sqrt{2\pi}} \exp\left(-\dfrac{1}{2}x^2\right)$ is the standard Gaussian density function. The asymptotic expression (5.2) for $\bar{F}_L(x)$ is interesting by being independent of the distribution of $\tilde{X}$. Solely the mean μ and the coefficient of variation V of $\tilde{X}$

determine the limiting distribution. However, in order to check the acceptance criterion $\overline{F}_L(x) \le A(x)$ for all x by use of (5.2) it is necessary to determine the integral by numerical integration.

6. Several adverse event categories

The problem does not become easier when more than one adverse event category are relevant. There are two ways of analysis. One way is to use that the superposition of a finite number n of mutually independent Poisson point processes with intensities $\lambda_1, \lambda_2, \ldots, \lambda_n$ is a Poisson point process with intensity $\lambda = \lambda_1 + \lambda_2 + \ldots + \lambda_n$. Accordingly the accumulated loss process $L(t)$ can be defined on the basis of the Poisson process of intensity λ by assigning a loss to each adverse event occurrence which is distributed as an inhomogeneous random variable X with the mixed distribution function

$$F_X(x) = \frac{1}{\lambda} \sum_{i=1}^{n} \lambda_i F_{X_i}(x) \qquad (6.1)$$

(the index i now referring to the category number). Then the asymptotic result (5.2) is still valid with

$$E[X] = \frac{1}{\lambda} \sum_{i=1}^{n} \lambda_i E[X_i] \qquad (6.2)$$

$$1 + V_X^2 = \frac{1}{\lambda} \sum_{i=1}^{n} \lambda_i (1 + V_{X_i}^2) \qquad (6.3)$$

to be used in (4.4) and (4.5) to calculate $E[\tilde{X}]$ and $V_{\tilde{X}}$ However, if the degree of inhomogeneity of X is large [an imprecise statement that intuitively means that there (expressed by another imprecise statement) is a large spread of the pairs $(E[X_1], D[X_1]), \ldots, (E[X_n], D[X_n])$], then it should be expected that the convergence to the asymptotic expression (5.2) is slow as $x/\mu \to \infty$. The asymptotic expression (5.1) may not be directly applicable because λ need not be sufficiently small to justify the omission of the higher order terms.

The other way of analysis is to use that the accumulated loss $L(t)$ can be written as $L(t) = L_1(t) + \ldots + L_n(t)$ where $L_1(t), \ldots, L_n(t)$ are the accumulated mutually independent losses for each of the categories of adverse events. Thus the problem is that of calculating the distribution function for the sum of n differently distributed independent random variables. For exact analysis this requires n successive convolution integrations of the relevant distributions. However, approximate calculations of $\overline{F}_L(x)$ can be made for each x (x large) much easier by use of FORM or SORM (first or second order reliability methods) available in several commercial software packages (e.g. PROBAN, DNV or STRUREL, RCP, see Ditlevsen and Madsen (1996)).

7. Approximate risk profiles

An operational approximate procedure is to adopt the distribution type (5.2) as the distribution type for $L(t)$ with λT as given, but with the parameters μ, V of the distribution calibrated to values μ_c, V_c such that the distribution gets the exact expectation and square of the coefficient of variation

$$E[L(T)] = \lambda T \, E\left[\tilde{X}\right] \tag{7.1}$$

$$V_{L(T)}^2 = \frac{1}{\lambda T}(1 + V_{\tilde{X}}^2) \tag{7.2}$$

respectively. This requires an iterative solution procedure using numerical integration as explained in Ditlevsen (1995).

A simpler first check is to let the distribution of $L(T)$ be approximated by the lognormal distribution of mean given by (7.1) and coefficient of variation given by (7.2). This assumption of lognormal distribution is not based on any theoretical considerations but is solely a pragmatic choice. Graphical comparisons shown in Ditlevsen (1995) demonstrate that the lognormal distribution makes a good fit except in the upper tail of very small probabilities where it is conservative as compared to the fitted asymptotic distribution.

8. Authority restricted decision making

Decision making based on the decision axioms of Von Neumann and Morgenstern (1943) requires that the decision maker can assign relative preference weights (so-called utilities) to all the possible consequences of all the acts that can possibly result from the decision. In principle these assignments reflect the personal preferences of the decision maker. There may be a large difference between the utility of a given consequence as assigned by a risk averse decision maker and that assigned by a risk prone decision maker. For decision problems of societal impact, e.g. decisions concerning projects or operations that may endanger human lives, threaten the environment, or involve other values of concern of the society, it is therefore necessary that some superior authority specifies some restrictions on the assessment of the utilities.

The decision theory is about situations where only one of the possible consequences of an act occurs when the act is realised. Thus each act that can be chosen by the decision consists in running a lottery between the consequences. Each lottery is characterised by a set of probabilities distributed over the consequences. The set of realisable lotteries is a subset of the set of all lotteries between the consequences. According to the decision axioms of Von Neumann and Morgenstern it is then the best decision to choose the act that realises the lottery of maximal expected utility among the releasable lotteries.

In the past the standard way of formulating authoritative restrictions has been to specify upper bounds on the probabilities of getting certain types of adverse events within a given period of operation. This type of restriction can be interpreted as a restriction on the set of realisable lotteries in the decision problem. A more rational way, it seems, is that the authority specifies lower bounds on the socio-economic loss values (negative utility values) for the adverse events that are intangible in ordinary economic terms. These socio-economic values should be specified on some monetary scale such that all consequences become commensurable on this scale.

Unrestricted decision optimization then applies in a rational way with the consequence that the optimal probabilities of the adverse events will vary from case to case. In practice this way of making authoritative restrictions has a similar society protecting effect as the restrictions on the probabilities. However, irrespective of how large a value is assigned to a human life it may by some people be considered as an unethical principle to weigh human lives economically against economical benefit. Of some mysterious reason it seems as if it is easier for the public to accept bounds on the probabilities even though these bounds are just as arbitrary as the socio-economical value settings. As mentioned in the introduction the bounds are often obtained simply by accepting the risk associated with the practice of the past.

The problem about which probability bound values the authority should specify can be viewed as a decision optimization problem, with the superior authority as the decision maker. The adverse event probability bound will cause a loss each time the bound becomes active. If the authority has chosen the socio-economic values for its own considerations, the authority can apply an unrestricted decision optimization. It is in principle possible for a specified set of operations or projects to calculate the overall cost of the restriction as a function of the bound. Besides the saved socio-economic values (that by mathematical necessity cannot by itself overweigh the monetary losses caused by the restriction) the counteracting gain is a benefit for the authority of avoiding public reactions on the occurrence of adverse events. The value of this benefit increases with the decrease of the number and seriousness of such complaints, that is, the benefit increases with decreasing probability bounds. Thus the authority may have an optimal choice of the probability bounds.

This decision making scheme is helpful for the mind, but hardly easy to carry out in practice. Therefore the probability bounds are usually specified by the authority directly on the basis of statistics on experienced adverse events in connection with similar types of human activity. The observed occurrence rate level seems to be identified with the level tolerated in the past.

9. Risk aversion regulated by consequence dependent acceptance bounds

As mentioned in the introduction the acceptance criteria formulating authorities may be concerned about the phenomenon of risk aversion showing up as more severe public reactions to a large number of casualties by a single rare accident than to the same accumulated number of casualties distributed over several accidents. In other words, the authorities tend to have an acceptance policy where the expected loss [obtained from (7.1) and (4.4)]

$$E[L(T)] = E[X]\frac{\lambda}{\gamma}(1-e^{-\gamma T}) \qquad (9.1)$$

for each homogeneous category of adverse events is accepted only if the occurrence rate λ is below a bound which decreases faster than $1/E[X]$ for increasing $E[X]$. This should reflect that there is an additional expected loss beyond $E[X]$ imposed on the public, and this additional loss is the larger the larger $E[X]$. This way of authority handling of the risk aversion phenomenon, the existence of which cannot be denied, is somewhat mysterious in the sense that it is difficult to see a rational principle of decision making as the basis for it. An objection against the procedure is that even if the owner company of the device makes a qualified risk analysis and obeys the acceptance criteria, the experience shows that the company will often get an authority imposed extra penalty (e.g. in the form of more strict future operation conditions) after the occurrence of a larger adverse event. This extra penalty is on top of the direct loss and the general cost of keeping the required bound on the occurrence rate λ. It is even so that several other companies running the same type of operation will be penalised simultaneously with the company that had the adverse event. Assuming that the acceptance criteria making authority has a rational principle for setting the acceptance bound under consideration of risk aversion, it is difficult to understand why the authority should impose such an extra penalty after the occurrence of an adverse event of a large loss. Societal risk aversion of the type considered here is an irrational phenomenon, but it should not of this reason be neglected by the decision maker. The question is alone whether it is possible to take care of the irrationality in a rational way. This problem is considered in the light of the decision theory in Section 10.

Obviously risk aversion motivated bounds only serve their intended purpose if they are imposed on the occurrence rate λ for each single adverse event category sequence. Thus the acceptance bounds $A(x)$ and $B(x)$ imposed on the total risk profile $\overline{F}_L(x)$ may become supplemented by one or more authority imposed acceptance bounds on the occurrence rate. These may differ from category to category. For example, for the same number of casualties, hotel fires may be considered worse than air plane crashes.

A practical example of acceptance bound setting in dependence of the number of fatalities by a single adverse event is given in the document : Risk Acceptance Criteria for Design of the Storebælt Link (1990). For the entire link the central estimate of λ must be bounded by $2\cdot 10^{-2}, 3\cdot 10^{-5}, 3\cdot 10^{-6}$ per year for the number of fatalities being 1-19, 20-200, > 200, respectively. Clearly these numbers reflect risk aversion against public reactions.

10. Risk aversion considered in its proper decision context

It is a fact that after the occurrence of a larger rare adverse event the public tends to react against the current practice of running an operation or making project decisions. This is a kind of paradox when considering that the public without reactions accepts a sequence of minor adverse events that during the waiting time to the occurrence of the larger adverse event may accumulate even a larger number of fatalities.

As discussed in an earlier section it has sometimes been the attempt of the authority to formulate the acceptance criteria under consideration of this risk aversion phenomenon even though the authority lacks a rational quantitative basis for doing so. It is difficult to use the same kind of philosophy as discussed in the previous section because the loss triggered by the public reaction is not taken by the authority (the society) in a clearly disproportionate degree relative to the societal losses imposed by all other types of occurrences of adverse events. The politicians currently possess the authority of setting the acceptance criteria. A motivation for their risk aversion might be a fear of being claimed responsible for setting insufficiently strict acceptance criteria in case the large adverse event should occur. However, it seems difficult to defend that all larger operations and projects going on in the society should be loaded with this extra cost of irrational origin in order to protect the goodwill of the politicians. Moreover, the adverse events considered here are controlled to be of so rare occurrence that if such an event occurs at all, the time of occurrence will most likely be when the political authority is no longer in the hands of those politicians that specified (or ratified) the acceptance criteria.

Instead of being a concern of the authority the risk aversion phenomenon should reasonably be taken into consideration by the responsible owner of the device or the operation business. The owner should be aware that the public reaction on a larger adverse event may cause the authority to impose restrictions on the future operation or require that safety improving technical remodelling of the device be made. This consequence of a public reaction can be cost evaluated on a rational engineering basis and be included in the decision making of the owner.

Consider a category of adverse events with random losses of a size that possibly may give public reactions that trigger the introduction of future restrictions. Assume that the Poisson model can be applied for the sequence of these adverse events, and assume as an example that the first time the loss X exceeds the critical level ξ, then the restriction is imposed with the purpose of reducing the occurrence rate from λ to $\lambda' < \lambda$. Then it can be shown, Ditlevsen and Madsen (1996), that the total expected loss becomes

$$E[L(T)] = \frac{\lambda}{\gamma}\frac{E[X](\gamma + \lambda' p) + ap}{\gamma + \lambda p} \qquad (10.1)$$

asymptotically as $T \to \infty$, where $p = \overline{F}_X(\xi)$. Compared to the asymptotic limit $E[X]\,\lambda / \gamma$ obtained from (8.1), this expression contains the yearly cost a of the increased operation control imposed at the time of the first occurrence of $X > \xi$. In practical applications it is ordinarily so that $\lambda p / \gamma << 1$, and the total expected loss may be simplified to

$$E[L(T)] = \frac{\lambda}{\gamma}\left[E[X] + \frac{ap}{\gamma}\right] \qquad (10.2)$$

showing that the phenomenon of risk aversion is rationally taken care of simply by increasing the expected loss $E[X]$ of a single adverse event by the expected cost ap / γ of the political intervention given that the adverse event occurs. With the proper interpretation of the cost value a, this asymptotic formula for the expected loss is valid also for restrictions that require a remodelling of the device rather than a change of the operation conditions in the future. It is emphasised that the value of life and limb is included in X and not in a / γ. Thus the aversion cost depends on the type of imposed restriction and not on the number of casualties except for the public reaction threshold ξ. The expected cost values a for the technically possible restrictions can be evaluated at least in terms of probability distributions by rational engineering analysis and modelling, while information about the threshold ξ may be obtained from statistics on past experienced cases. Also ξ may be modelled as a random variable, but then an explicit expression of the total expected loss can possibly not be obtained. In such a case the expected loss can be assessed numerically by straight-forward simulation studies.

11. Capitalization of life and limb to present value

It may be considered as being an unethical principle to capitalise human lives by applying an interest rate $\gamma > 0$ on the value of a human life, that is, to assign the formal value $X\exp[-\gamma t]$ at time zero to X human lives lost at time t. However, the following example illustrates the mathematical inconvenience of counting fatalities without using capitalization in time with a positive interest rate γ.

Consider an operation that produces a net capital gain of g per time unit but also has adverse events with casualties according to a Poisson process of intensity λ. This intensity can be controlled at a cost $c(\lambda)$ invested at time zero. The net loss of the operation during time T capitalized to time zero is then

$$c(\lambda) + \sum_{n=1}^{N(T)} X_n e^{-\gamma_1 \tau_n} - g\int_0^T e^{-\gamma_2 \tau} d\tau \qquad (11.1)$$

where $X_1,\ldots,X_n,\ldots$ are the independent and identically distributed values of the casualties at times $\tau_1,\ldots,\tau_n,\ldots$, respectively, and γ_1,γ_2 are the interest rates on the casualties and the monetary gains, respectively. By use of (8.1) it follows that the expected value of (11.1) is

$$c(\lambda) + E[X]\frac{\lambda}{\gamma_1}(1-e^{-\gamma_1 T}) - \frac{g}{\gamma_2}(1-e^{-\gamma_2 T}) \rightarrow c(\lambda) + E[X]\frac{\lambda}{\gamma_1} - \frac{g}{\gamma_2} \qquad (11.2)$$

as $T \rightarrow \infty$. Assuming that $dc(\lambda)/d\lambda < 0$, the expected loss has a minimum for

$$\frac{dc(\lambda)}{d\lambda} = -E[X]\frac{1}{\gamma_1} \qquad (11.3)$$

Thus no solution can be obtained for $\gamma_1 = 0$ if T is allowed to be infinite. For T fixed and $\gamma_1 \rightarrow 0$ the expected value of (11.1) becomes

$$c(\lambda) + E[X]\lambda T - \frac{g}{\gamma_2}(1-e^{-\gamma_2 T}) \qquad (11.4)$$

with minimum for

$$\frac{dc(\lambda)}{d\lambda} = -E[X]\,T \qquad (11.5)$$

It is seen that the optimal solution depends on the arbitrary choice of the reference time T.

The notion that it is unethical (or just unrealistic) to make interest rate calculations on casualties has been used as an argument for choosing $T = 1$ year for the acceptance probabilities related to casualties and $T =$ the entire planned operation time for the acceptance probabilities related to other values, AS Storebæltsforbindelsen (1990). However, by comparing (11.3) and (11.5) it is seen that in a decision theoretical concept the finite time T is completely equivalent to have the interest rate $\gamma_1 = 1/T$ applied during infinite time. For $T = 1$ year the equivalent yearly interest capitalization factor becomes so extremely large as $e \cong 2.27$. For the monetary values a reasonable interest capitalization factor per year is about $1.05 \cong \exp(0.05)$. Choosing $\gamma_1 = 0.05$ gives a factor $1/\gamma_1 = 20$ in (11.3). Thus the decision theoretical equivalent time without using capitalization of casualties should in this example be $T = 20$ years. This obviously leads to a lower optimal value of the adverse event rate λ than obtained for $T = 1$ year.

The conclusion is that effectively it may show more concern about the future loss of human lives to apply a reasonable interest rate of the same order of size as for the monetary values (cleaned for inflation, of course) on an infinite time horizon than only considering those lives that are lost within a specified duration T with zero interest rate on the values of these lives.

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