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Reliability analysis of large bridge box girder by model-correction-factor method

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ABSTRACT: The model-correction-factor method for reliability analysis has in previous papers been demonstrated to be very effective for small textbook type structural examples. In this paper the mechanical properties of a large steel box girder of an actual bridge project is modelled by a non-linear FE-program that in direct use makes any type of reliability analysis extremely computer time consuming. Thus limit state simplifying techniques have to be employed. The model-correction-factor method is based on a principle of replacement of a complicated limit state surface by an adapted limit state surface that originates from a suitably simple mechanical theory such as the rigid-plastic theory. The adaptation is made by applying a calculated correction factor (effectivity factor) to those material parameters of the problem that in their physical units contain the unit of force (yield strengths, for example) such that the two limit state surfaces give approximately identical failure probabilities. This correction factor is calculated by a single or some few applications of the FE-program. The input to the FE-program is determined by a first order reliability analysis on the basis of the simple model. It is found that also in this large structure example the model-correction-factor method is fast and accurate and that it by intelligent simplifying engineering modelling cuts the computer efforts drastically down.

1. INTRODUCTION

The fact that the development in the computer based structural analysis and design increasingly penetrates into engineering practice confronts the practical structural reliability evaluation with the problem of dealing with increasingly more complicated limit state formulations. Often the limit states become defined on the basis of such elaborate theoretical modelling that only numerical representations are feasible. As such they can often only be obtained by the use of time consuming solution procedures, for example based on finite element modelling or finite difference approximations. In each specific deterministic analysis made to guide the design process the actual computer program is usually only run in practice some few times, and therefore these programs almost as a rule become developed to the limit of acceptable time consumption for a single run given the available computer power. However, large time consumption for obtaining just a single point of the limit state surface is a problem in the structural reliability analysis irrespective of whether it uses the standard first or second order methods (FORM/SORM) or simulation methods.

The way to overcome this difficulty is to replace the complicated and usually implicitly defined limit-state surface by an approximating surface of much simpler description suited for practicable reliability analysis. It may be sufficient simply to replace the limit state surface by a second degree polynomial (or some other linear combination of simple functions) in the input variables and use standard regression technique to obtain a "best" fit on the basis of a limited number of calculations in the complicated model. However, if it is not known in advance where in the space of input variables the most important part of the limit state surface is situated it is a problem to select a suitable domain within which the approximation to the limit state surface is of sufficient accuracy. If the most central point on the response surface become situated outside this domain, a new surface must be fixed around this point. Unfortunately it is not sure that the most central point on the new surface is within the accuracy domain for this surface. Thus an iteration sequence is started for which convergence is not guaranteed.

The model-correction-factor method serves the same purpose as the so-called response surface methods, but in contrast to the standard response surface method the model-correction-factor method has the advantage of being supported on sound principles of mechanics both advancing its possibilities of fast convergence and improving the insight into the problem. A short description without proofs of the zero order version of the method is as follows. Let

$$g(\mathbf{x},\mathbf{y},\mathbf{z}) = 0,\ h(\mathbf{x},\mathbf{y},\mathbf{z}) = 0 \qquad (1,2)$$

be two limit state equations where $\mathbf{x},\mathbf{y},\mathbf{z}$ are the input vectors of load variables, resistance variables, and remaining geometric or dimensionless variables, respectively. All variables in $\mathbf{x}$ and $\mathbf{y}$ are assumed to contain the unit of force in their physical units.

Assume that the two limit states are such that there exists unique functions $\gamma(\mathbf{x},\mathbf{y},\mathbf{z})$ and $\eta(\mathbf{x},\mathbf{y},\mathbf{z})$ that make the two equations

$$g[\gamma(\mathbf{x},\mathbf{y},\mathbf{z})\mathbf{x},\mathbf{y},\mathbf{z}]=0,\; h[\eta(\mathbf{x},\mathbf{y},\mathbf{z})\mathbf{x},\mathbf{y},\mathbf{z}]=0 \quad (3,4)$$

into identities. Then it can be shown that it is a simple consequence of the dimension homogeneity of the two limit state equations that the function

$$\nu(\mathbf{x},\mathbf{y},\mathbf{z})=\gamma(\mathbf{x},\mathbf{y},\mathbf{z})\,/\,\eta(\mathbf{x},\mathbf{y},\mathbf{z}) \quad (5)$$

acts as a local effectivity factor when applied to the resistance variables in the limit state equation (2): $h(\mathbf{x},\mathbf{y},\mathbf{z})=0$. Thereby the limit state defined by the equation

$$h[\mathbf{x},\nu(\mathbf{x},\mathbf{y},\mathbf{z})\mathbf{y},\mathbf{z}]=0 \quad (6)$$

becomes identical to the limit state defined by the equation (1): $g(\mathbf{x},\mathbf{y},\mathbf{z})=0$.

If it is assumed that the function $h(\mathbf{x},\mathbf{y},\mathbf{z})$ is defined in such a wise way that it with respect to its gradient direction field has a reasonably close coincidence with the gradient direction field of the function $g(\mathbf{x},\mathbf{y},\mathbf{z})$ over a suitable neighbourhood of a locally most central point $(\tilde{\mathbf{x}},\tilde{\mathbf{y}},\tilde{\mathbf{z}})$ on the limit state surface $g(\mathbf{x},\mathbf{y},\mathbf{z})=0$, then the local effectivity factor $\nu(\mathbf{x},\mathbf{y},\mathbf{z})$ is close to be a constant within this neighbourhood. Actually, if it is assumed that $(\tilde{\mathbf{x}},\tilde{\mathbf{y}},\tilde{\mathbf{z}})$ is a point of stationarity of $\nu(\mathbf{x},\mathbf{y},\mathbf{z})$, then the two limit state surfaces

$$g(\mathbf{x},\mathbf{y},\mathbf{z})=0,\; h[\mathbf{x},\nu(\tilde{\mathbf{x}},\tilde{\mathbf{y}},\tilde{\mathbf{z}})\mathbf{y},\mathbf{z}]=0 \quad (7,8)$$

are tangential to each other at $(\tilde{\mathbf{x}},\tilde{\mathbf{y}},\tilde{\mathbf{z}})$, implying that the two limit state surfaces both have $(\tilde{\mathbf{x}},\tilde{\mathbf{y}},\tilde{\mathbf{z}})$ as a locally most central limit state point.

The following iteration procedure suggests itself. Starting with some judgmental choice of the value ν_0 of $\nu(\tilde{\mathbf{x}},\tilde{\mathbf{y}},\tilde{\mathbf{z}})$, a most central point $(\mathbf{x}_1,\mathbf{y}_1,\mathbf{z}_1)$ on the surface

$$h(\mathbf{x},\nu_0\mathbf{y},\mathbf{z})=0 \quad (9)$$

is determined together with an approximate failure probability p_1. Next the improved value

$$\nu_1=\nu_0\gamma(\mathbf{x}_1,\mathbf{y}_1,\mathbf{z}_1) \quad (10)$$

of $\nu(\tilde{\mathbf{x}},\tilde{\mathbf{y}},\tilde{\mathbf{z}})$ is calculated, whereupon a most central point $(\mathbf{x}_2,\mathbf{y}_2,\mathbf{z}_2)$ on the surface

$$h(\mathbf{x},\nu_1\mathbf{y},\mathbf{z})=0 \quad (11)$$

is determined together with an approximate failure probability p_2. Proceeding iteratively in this way we get a sequence

$$[\gamma(\mathbf{x}_1,\mathbf{y}_1,\mathbf{z}_1),p_1],[\gamma(\mathbf{x}_2,\mathbf{y}_2,\mathbf{z}_2),p_2],\ldots \quad (12)$$

that may or may not be convergent. If it is convergent in the first component it is also convergent in the second component, and we have that

$$\{(\mathbf{x}_i,\mathbf{y}_i,\mathbf{z}_i)\}\to(\tilde{\mathbf{x}},\tilde{\mathbf{y}},\tilde{\mathbf{z}}),\; \{\gamma(\mathbf{x}_i,\mathbf{y}_i,\mathbf{z}_i)\}\to 1,\{p_i\}\to p \quad (13)$$

where p is called the zero-order approximation to the failure probability corresponding to the limit state (1): $g(\mathbf{x},\mathbf{y},\mathbf{z})=0$. If the sequence is not convergent, the zero-order approximation can be obtained by interpolation to the value $\gamma=1$ among the points of the sequence. If $(\tilde{\mathbf{x}},\tilde{\mathbf{y}},\tilde{\mathbf{z}})$ is not a stationarity point of $\nu(\mathbf{x},\mathbf{y},\mathbf{z})$, the zero-order approximation may still be sufficiently accurate. A check can be made by replacing $\nu(\mathbf{x},\mathbf{y},\mathbf{z})$ by its first-order Taylor expansion at the obtained limit point $(\tilde{\mathbf{x}},\tilde{\mathbf{y}},\tilde{\mathbf{z}})$ followed by a first order reliability calculation. Proofs are given in (Ditlevsen and Arnbjerg-Nielsen 1994). The point $(\tilde{\mathbf{x}},\tilde{\mathbf{y}},\tilde{\mathbf{z}})$ is hereafter called the zero-order design point.

Simple rigid-plastic structural models turn out to define effective replacement surfaces for reliability analysis of different textbook types of examples of structural collapse behaviour that deviates from rigid-plastic mechanism behaviour (Ditlevsen and Arnbjerg-Nielsen 1994), (Johannesen and Ditlevsen 1993), and also for an evaluating the collapse probability according to the static theorem by use of the kinematic theorem, (Karlsson et al 1993).

In this paper the model-correction-factor method is applied to a large steel box girder of type as used for the approach spans to the eastern highway bridge across the Great Belt in Denmark. The girder is loaded by its self-weight and a discretized traffic load that corresponds to a vehicle queue formation on the girder. For a total collapse situation, a maximal displacement requirement, and a situation with buckling of the side panels at the supports, three different limit state surfaces are defined implicitly on the basis of calculations with a commercial non-linear finite element program (FENRIS, DNV). The corresponding simplified limit states are defined on the basis of ad hoc structural models each constructed to be as simple as judged to be reasonable under due consideration that each of the input variables grossly gets similar influence on the simplified limit states as on the given limit states within the input variable domain that contributes the most to the probability of violation of the actual limit state. As in the previous papers it is shown herein that for both the collapse limit state and the buckling limit state it is sufficient to let the ad hoc models be of rigid-plastic constitutive behaviour while the displacement limit state only needs a simple non-linear rotation-spring hinge model.

Only zero-order approximations are reported herein. Checks for approximate tangential coincidence are made by directional simulation concentrated in the direction of the zero-order design point. The samples are necessarily of limited in size (15 to 20 sample values) due to the considerable computer time needed to generate each single sample value by the FE-program. Thus the estimated confidence intervals are rather wide. The zero-order approximations are obtained by a computer effort that amounts to the generation of at most two to three sample values, and they are mostly deviating by less than 1% from the simulation results in terms of the reliability index $\beta = -\Phi^{-1}(p_f)$, where p_f is the failure probability.

2. BRIDGE GIRDER FE-MODEL FOR DISPLACEMENT AND COLLAPSE

The structure is a continuous steel plate box girder over a large number of spans each of length $L = 193$ [m]. The cross section is trapezoidal with a detailed geometry and dimensions as shown on Figure 1. Besides the everywhere acting self-weight, it is assumed that a single central span in all lanes is totally loaded by dense queues of vehicles, Figure 2 . In order to limit the size of the FE-model, only the half part of the traffic loaded span to the right of the support B is subdivided into a fine mesh of finite elements while a more coarse mesh is used in the entire span to the left of the support B. At the support A, the boundary condition is as an approximation formulated by specifying a linear stress distribution over the cross-section for which the resulting bending moment equals the bending moment in a corresponding linear elastic Euler-Bernoulli beam of infinitely many spans subjected to the same load as the given beam. (A more correct formulation is to model the bending moment reaction of the girder structure to the left of the support A as the reaction from a rotation spring. However, this is an unimportant issue for the study reported in this paper).

The steel material is assumed to be linear-elastic-ideal-plastic according to the von Mises yield condition with deterministic elasticity modulus $E = 2.1 \times 10^{11}$ [N/m^2] and random yield stress f_y. The complicated structural details of the real girder are replaced by simpler "equivalent" elements in the FE-model by and large as follows: the through stiffened outer skins and central longitudinal bulkhead are modelled as piece-wise homogeneous massive plates of the same local axial and bending stiffness in the longitudinal direction as the real through stiffened plates within the domain of elastic behaviour and with the same uniaxial yield strain $\varepsilon_y = f_y/E$; as in the real structure stiffening transverse combined frames and trusses are placed for each 4 [m]; a transverse bulkhead of infinite stiffness is placed over the supports; at each support four longitudinal beams stiffen the bottom plate.

3. VEHICLE QUEUE LOAD MODEL

The girder is assumed to carry 4 random line load fields placed along the centre lines of the 4 traffic lanes on the bridge. Each line load field is piece-wise constant with jumps for each $a = 8$ [m] taking place over every second transverse stiffening truss. Thus in total 4 times 12 random variables define the 4 line loads acting on half the span. These 48 random variables are assumed to be mutually independent with identical normal distribution of mean μ_p and variance σ_p^2. For the considered limit states these line load fields are reasonably representable models of the load from a single realisation of dense vehicle queues extending over the entire span in all 4 lanes. The parameters μ_p and σ_p^2 can be related to the parameters of the vehicle populations such as the percentage f of trucks among all vehicles in the queues, the truck and car occupation lengths (e.g. set to $2l$ and l, respectively), and the distribution parameters of the population of the truck weight W. In fact, if a linear load effect defined by an influence function $I(x)$ is considered, and the line load is modelled as a homogeneous white noise field, then the load field acting in the infinitesimal interval $[x, x+dx]$ gives an independent load effect increment of mean and variance

$$vH\, I(x)dx, \quad vK\, I(x)^2 dx \tag{14,15}$$

respectively, where

$$v = \frac{1-f}{(1+f)l} \tag{16}$$

is the mean number of cars per length unit of the queue

$$H = V + E[W]\frac{f}{1-f} \tag{17}$$

with V being the mean car weight, and (in its simplest form)

$$K = \frac{f}{1-f} Var[W] + \frac{f}{(1+f)^2} E[W - 2V]^2 \tag{18}$$

These formulas are obtained from a traffic stream composition model (Ditlevsen and Madsen 1994). For any influence function that only varies modestly over the interval of length a with constant load intensity the parameters μ_p and σ_p^2 can be directly calibrated to this white noise model by setting

$$\mu_p = vH, \quad \sigma_p^2 = vK/a \tag{19}$$

4. COLLAPSE AND DISPLACEMENT CRITERIA AND THE CORRESPONDING SIMPLE AD HOC MODELS

Obviously the simple ad hoc model for the collapse limit state is reasonably chosen as the rigid plastic beam mechanism model shown in Figure 2. This model is also a reasonable choice of the ad hoc model for the centre point displacement limit state except that the plastic hinges are replaced by non-linear rotation spring hinges.

For both the FE-model and the simple model the limit state function is put on the form

$$g(\mathbf{x},\mathbf{y},\mathbf{z}) = \lambda(\mathbf{x},\mathbf{y},\mathbf{z}) - 1 \qquad (20)$$

where $\lambda(\mathbf{x},\mathbf{y},\mathbf{z})$ is the load factor to be applied to the realisation $\mathbf{x}$ of the vehicle queue load such that the load $\lambda(\mathbf{x},\mathbf{y},\mathbf{z})\mathbf{x}$ causes the structure to be in the collapse state or have its centre point displaced a specified vertical length.

In the FE- model the displacement load factor is calculated by use of a Newton-Raphson algorithm. The load factor λ is increased in small increments until the limit displacement is exceeded without violating an upper limit for the step length in a number of the preceding load increments. From the last three load factor values versus displacements, the load factor corresponding to the displacement limit is calculated using a second order polynomial fit.

The collapse load factor λ is in the FE- model defined as the maximal obtainable factor when assuming proportional loading after application of the self-weight load. An arc-length control algorithm (Bjærum 1992) is used for the vehicle queue load incrementation and the equilibrium iteration. Each calculation run is stopped when the load factor maximum value is passed without violating an upper limit value for the absolute increment sizes in the vicinity of the maximum. The arc-length control algorithm can detect bifurcation points, i.e. points with more than one incremental displacement direction in equilibrium, but bifurcation points have not been detected in the present problem. It should be pointed out, however, that a detailed study of the bifurcation phenomenon in relation to the considered FE-meshing has not been carried out for the collapse failure criterion because buckling is unlikely to occur for the given structural geometry.

With respect to the sensitivity of the results to the choice of the finite element mesh a number of calculations has shown that the relative difference between the load factor values for the used mesh and a mesh with roughly half the number of shell elements in the longitudinal and the transverse direction is of the order 5 %.

For the simple plastic model the maximal queue load factor λ is explicitly obtained by equating the external work and the plastic dissipation:

$$\frac{1}{12}\lambda(6\mu_p + \sigma_p U)L^2 = M_{ys} + M_{yc} - \frac{1}{8}p_g L^2 \qquad (21)$$

where U is a standard normal random variable, M_{ys} and M_{yc} are the yield moments of the support and the centre cross-section, respectively, and p_g is the self-weight load per unit length of the girder set to a constant equal to the average over the span length L. The cross-section yield moments are calculated by use of the correct plate cross-sections.

The rotation springs of the ad hoc displacement model are modelled by the constitutive relation

$$\theta = \frac{M}{k}\left[1 + c\frac{M}{M_y}\tan\left(\frac{\pi M}{2M_y}\right)\right] \qquad (22)$$

between the relative angular rotation θ and the spring moment M, Figure 4. The initial stiffness k and the yield moment M_y are put to k_s and M_{ys}, respectively, for the support springs, and to rk_s and M_{yc}, respectively, for the centre spring, where r is the ratio between the moments of inertia about the horizontal principal axis of the centre and support cross-section of the bridge girder. The value of k_s is chosen to be the one obtained for $c = 0$ by setting the vertical displacement of the centre point equal to the corresponding displacement for a linear-elastic Euler-Bernoulli beam model of the bridge girder with the same uniformly distributed load on the two models. Only a crudely chosen value of the parameter $c \geq 0$ is needed and a single value for c has been found to be applicable in all cases. The factor λ on the vehicle queue load corresponding to the required displacement limit is then easily calculated by use of the geometry constraints for the rotations, the constitutive relations for the springs, and the moment equilibrium equation for the considered half span.

The sufficiency of only considering the left half of the loaded span follows from splitting the line load field $p(x)$ in the symmetric part

$$\frac{1}{2}[p(x) + p(L-x)] \qquad (23)$$

and the anti-symmetric part

$$\frac{1}{2}[p(x) - p(L-x)] \qquad (24)$$

The two parts are uncorrelated and both have the variance $\sigma_p^2/2$. The displacement field and the collapse mechanism symmetry implies that the anti-symmetric part has no influence except for some negligible effects caused by the randomness of the the material properties.

5. BUCKLING FAILURE CRITERION AND THE CORRESPONDING SIMPLE AD HOC MODEL

As a third example a limit state of buckling of the inclined plate walls near the support is considered. The actual through stiffened girder walls are too bending stiff to make elastic buckling of any panels occur before plastic collapse occurs. For the present purpose of testing the model-correction-factor method for reliability analysis all through cross-sections and plate thicknesses are therefore reduced by a factor of 0.25. The buckling analysis requires a finer FE-mesh than used in the collapse and displacement analyses. Only the part of the girder that ranges over the six plate fields (24 [m]) nearest to the support is subdivided into finite elements. The transverse bulkhead over the support is assumed to be infinitely rigid and undisplaceable. The cross-section 24 [m] out in the span is assumed to have linearly varying normal stresses and uniformly distributed shear stresses in the inclined side plates and in the vertical

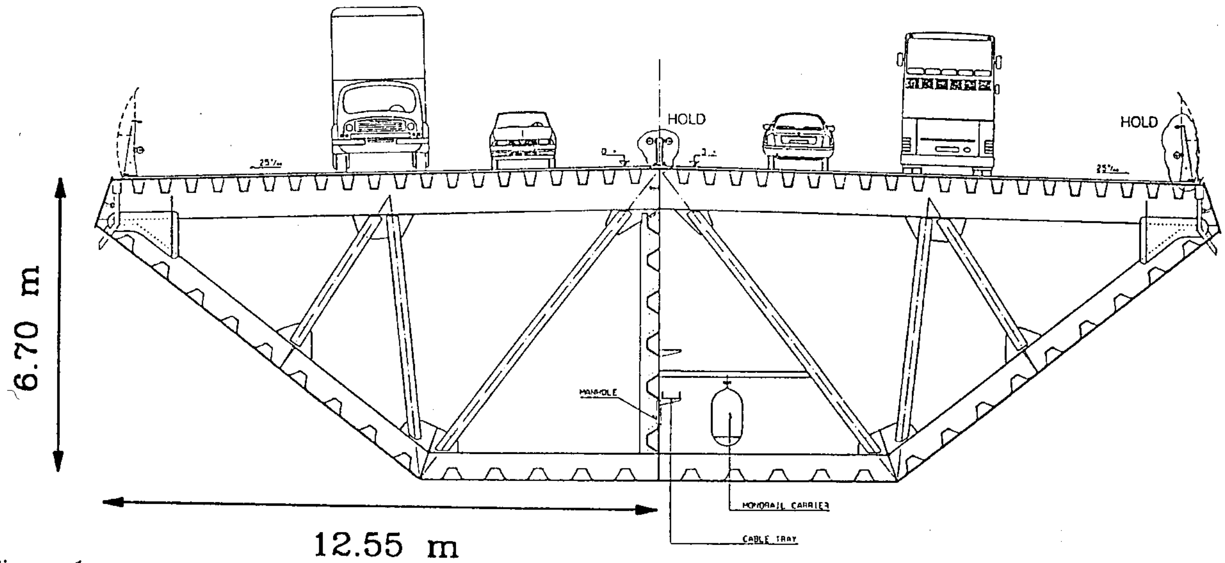

Figure 1.
Steel box girder cross-section (approach spans of the eastern Great Belt bridge).

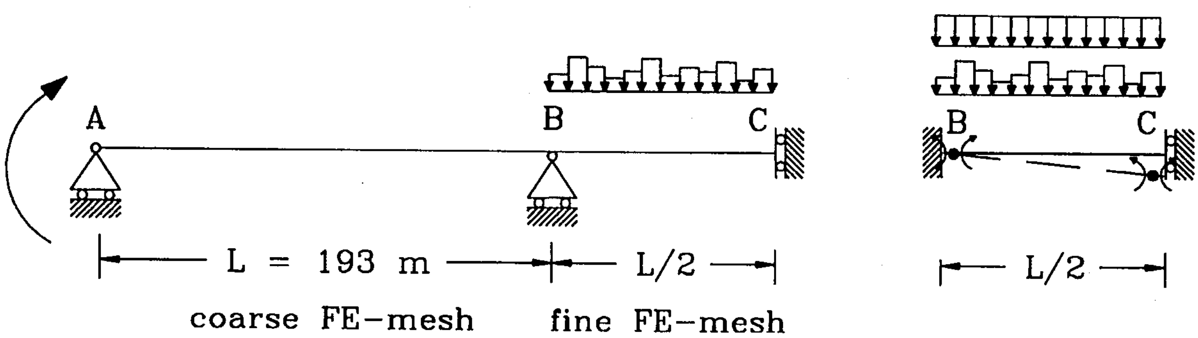

Figure 2.
Left: Static system-simplification of continuous bridge girder over several spans with indications of the vehicle queue load case and the two domains of the different FE-mesh subdivisions (Figure 3).

Right: Simple ad hoc model for both the displacement limit state and the collapse limit state chosen as a rigid beam with hinges.

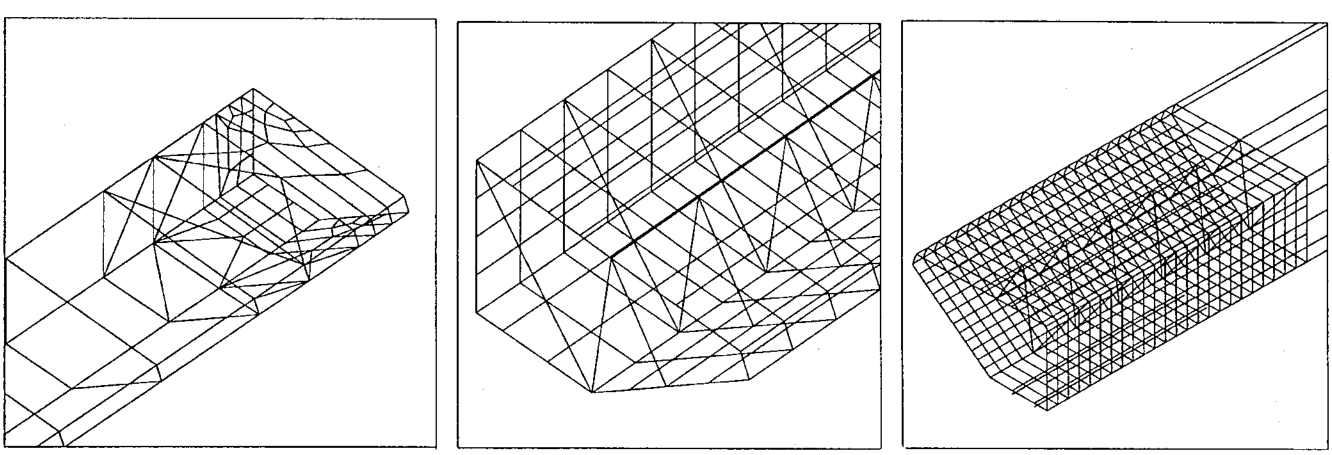

Figure 3.
FE-mesh as generated by the SESAM preprocessor for span A-B (centre). This mesh is used both for the displacement and the collapse limit state. The FE-mesh to the right shows the shell elements for the piece of the girder to the right of support B applied for the buckling limit state. Due to the symmetry only the right or the left parts of the FE-mesh are shown.

central bulkhead. The total stress distribution is required to be statically equivalent to the bending moment and the shear force obtained by continuing the girder to the centre point as a linear-elastic Euler-Bernoulli beam model with the a support at the centre as shown in Figure 2.

In a single calculation a relative difference of 8% has been found between the buckling load parameter for the chosen FE-mesh and a mesh with four times fewer shell elements to model the inclined and vertical plate fields. In a number of calculations it has been found by visual inspection of the buckling modes that the buckling occurs in the inclined plate fields at the support. This finding may not be general because the plate stiffness varies along the girder with highest stiffness at the support.

In the FE-model the buckling load factor λ_b is defined as the smallest value of λ for which the tangent stiffness matrix $\mathbf{K}(\lambda)$ becomes singular assuming that all stresses are within the elastic domain. It turns out that $\mathbf{K}(\lambda)$ varies almost linearly with λ so that it is sufficient to base the buckling calculation on the approximation

$$\mathbf{K}(\lambda) \approx \mathbf{K}(0) + \lambda \left(\frac{\partial \mathbf{K}(\lambda)}{\partial \lambda} \right)_{\lambda=0} \tag{25}$$

and with the derivative evaluated as the difference quotient using an increment in λ of about 5 % of λ_b. For simplicity the self-weight has been neglected in this example.

The ad hoc model is chosen as a simple rigid ideal plastic truss structure with the same yield strength of all the bars, Figure 5. The vehicle queue load is represented in statically equivalent form by the concentrated vertical forces applied at the joints in the upper flange. The forces applied in the free end are statically equivalent to the stress distribution applied in the FE-model in the corresponding cross-section. Available software (Damkilde et al 1994) for plastic analysis based on the lower bound theorem has been used for the truss collapse calculations.

Considering the unrealistically reduced dimensions, the boundary conditions including the loads are specified in this buckling example without attempts to achieve the most reasonable FE-model approximation to the static behaviour of the bridge girder when loaded by random vehicle queues that cover the entire most critical interval of influence on the shear forces near the supports, given the level of complexity of the FE-modelling.

6. DIRECTIONAL SIMULATION CONTROL

It is necessary to investigate whether the zero-order design point $\mathbf{p}_d$ besides being a locally or globally most central limit-state point for the simple ad hoc model also is a point which is situated sufficiently close to a locally most central point on the limit state surface of the FE-model. In other words, it should be checked whether the two limit state surfaces are approximately tangential to each other at the zero-order design point. A check via the first-order effectivity factor approximation is a possibility but rather time consuming due to the necessary gradient calculations and the large number of random variables in the problem. A practicable way is to perform directional simulation using a suitably concentrated bundle of random directions that in the mean sights towards the zero-order design point. In this study the importance of the uncertainty of the resistance variables is negligible as compared to the importance of the vehicle load variables. Therefore only the $n = 48$ independent and identically distributed normal random variables that define the complete queue load field have been included in the simulation. Following (Ditlevsen et al 1990) first an outcome $\mathbf{z}/\sigma_p$ of a random vector $\mathbf{Z}/\sigma_p$ with n-dimensional standard Gaussian distribution centred at the zero-order design point is generated. Next the load factor $\lambda(\mathbf{z})$ for which $\lambda(\mathbf{z})\mathbf{z}$ is a point on the limit-state surface for the FE-model is calculated. Finally the sampling variable value is calculated as

$$p = \frac{\varphi\left(\frac{\sqrt{n}\mu_p}{\sigma_p}\right)\varphi\left(\frac{\mathbf{z}'\mathbf{p}_d}{|\mathbf{z}|\sigma_p}\right)G\left(\frac{\lambda(\mathbf{z})|\mathbf{z}|}{\sigma_p}, \frac{\mu_p \mathbf{z}'\mathbf{e}}{|\mathbf{z}|\sigma_p}\right)}{\varphi\left(\frac{|\mathbf{p}_d|}{\sigma_p}\right)\varphi\left(\frac{\mu_p \mathbf{z}'\mathbf{e}}{|\mathbf{z}|\sigma_p}\right)G\left(0, \frac{\mu_p \mathbf{z}'\mathbf{e}}{|\mathbf{z}|\sigma_p}\right)} \tag{26}$$

$$G(x,y) = \sqrt{2\pi}\int_x^\infty s^{n-1}\,\varphi(s-y)\,ds \tag{27}$$

where $\varphi(x)$ is the standardised normal density function, and $\mathbf{e}' = [1\,1 \ldots 1]$. The average of the sample $p_1, \ldots, p_N$ obtained by N independent repetitions of this simulation calculation is an estimate of a number that approximately is the probability on the half-space outside the tangent hyper plane to the FE-limit-state surface at the zero-order design point. Relying on asymptotic normality of the estimator, confidence intervals are easily calculated.

7. RESULTS

The vehicle queue load data are based on data in (Ditlevsen et al 1994). Different queue load levels are specified in terms of a scaling factor on the mean lorry weight $E[W] = 188$ [kN], which is a value estimated for trucks in traffic in Denmark. It is noted that the queue line load mean value μ_p calculated according to Section 3 with $V \approx 10$ [kN] is approximately proportional to the scaled mean lorry weight. The coefficient of variation σ_p/μ_p is in dependence of the scaled mean lorry weight of the order of 1.6 to 2 in all examples. The yield stresses at different places of the girder are modelled in terms of 18 mutually independent normal random variables with mean 400 [Mpa] and standard deviation 25 [Mpa].

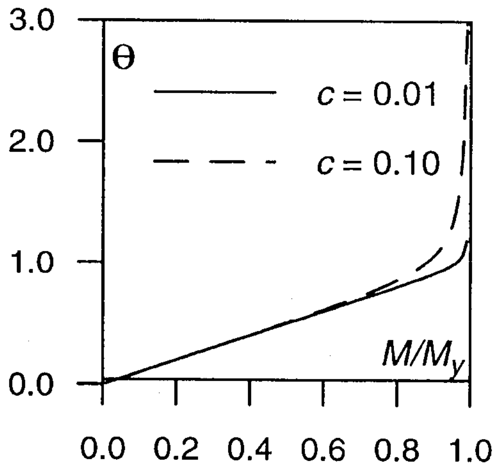

Figure 4.
Constitutive relation for the non-linear spring applied in the simple ad hoc model for the displacement limit state.The graphs show examples of the relative angular rotation θ versus the relative moment M/M_y for the initial stiffness $k=1$.

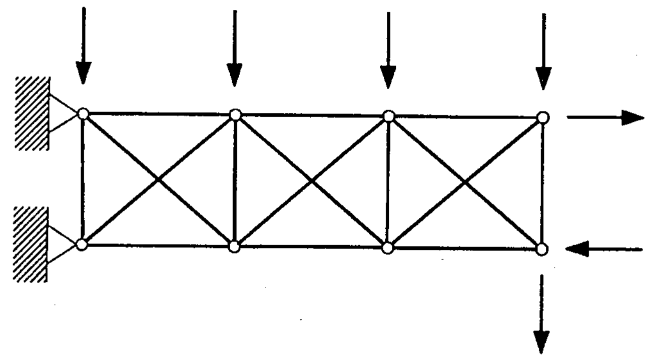

Figure 5.
Simple ad hoc model for the buckling limit state chosen as an ideal plastic truss structure.

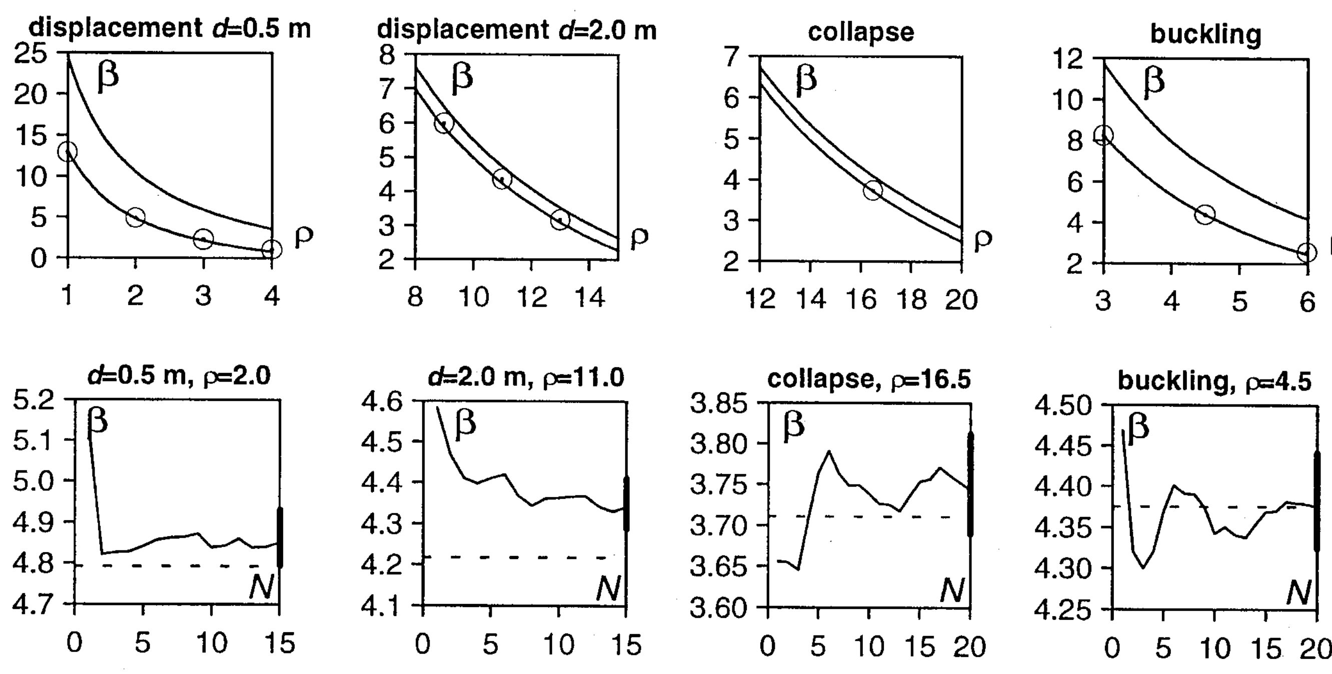

Figure 6.
Top diagrams: Reliability index β (FORM) versus load scaling factor ρ defined such that the mean truck weight is $\rho E[W]$ where $E[W]=188$ [kN]. The upper curves in these diagrams correspond to the simple ad hoc model without correction. The lower curves show the zero-order approximations obtained by the model-correction-factor method. The points in circles are the estimates obtained from the FE-models by use of directional simulation. Bottem diagrams: The estimates of β obtained by directional simulation directed towards the zero-order design point as a function of the sample size N. The vertical bars indicate the 90 % confidence intervals at the largest sample size. The zero-order approximations to β are shown as dashed lines.

The starting value of the effectivity factor is set to $\nu_0 = 1$ and the iteration stop criterion is chosen to

$$|1 - \gamma(\mathbf{x}_i, \mathbf{y}_i, \mathbf{z}_i)| < 0.01 \tag{27}$$

(Section 1). Using an HP9000/735 computer the CPU time for a single FE-calculation in dependence of the number of necessary load increments ranges from about 5 minutes for a buckling calculation, a fraction of an hour for a displacement calculation, and one to two hours for a collapse calculation. Due to computer share, the practical time was much larger. In contrast, for the simple models using the commercial program PROBAN from DNV, the CPU times needed for a complete FORM reliability analysis including the calculation of all sensitivity factors is negligible as compared to a single deterministic FE-calculation. Therefore only the number of invocations of the FE-program are reported in the examples.

Collapse limit state: The zero-order reliability indices obtained by the model-correction-factor analyses and the corresponding directional simulation estimates are shown in Figure 6. Only two FE-calculations were necessary to satisfy the stop criterion. The calculated very large reliability index values are explainable by the fact that the decisive design criterion for the girder concerns comfort limitations of the vibration accelerations induced by wind.

Displacement limit state: The analyses have been carried out for the centre point displacement limits of 0.5 [m], 1.0 [m], and 2.0 [m]. Yielding is in some cases observed to occur before passing the third limit, but not before passing the second limit. Only 2 or 3 FE-calculations were necessary to satisfy the stop criterion. The zero-order reliability indices and the corresponding directional simulation estimates are shown in Figure 6 for the displacements limits 0.5 [m] and 2.0 [m]. It is seen that the zero-order approximations are in good coincidence with the simulation results, even for the 0.5 [m] displacement limit for which there are relatively large differences between the reliability indices for the non-corrected and the corrected simple model.

Buckling limit state: The elastic properties are modelled to be deterministic. The zero-order reliability indices and the corresponding directional simulation estimates are shown in Figure 6.

Sensitivity analysis: The investigations also show that the effectivity factors are insensitive to parameter variations. Therefore the sensitivity factors obtained from the simple models at the zero-order design points are directly applicable to assess the reliability index values after specified parameter changes. In fact, without affecting the purpose of this investigation the reported reliability index results for the collapse and the displacement limit states have erroneously been calculated without the factor 1/2 on the variance σ_p^2 (see end of Section 4). Recognising this error in the input of the load variance, an approximate correction of the reliability index is easily made by applying the corresponding sensitivity factor to the correcting increment in the variance. This correction makes the reliability level of the bridge girder even larger than shown in Figure 6.

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