System Reliability Analysis by Model Correction Factor Method

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ABSTRACT: The Model Correction Factor Method is an intelligent response surface method based on simplified modeling. MCFM is aimed for reliability analysis in case of a limit state defined by an elaborate model. Herein it is demonstrated that the method is applicable for elaborate limit state surfaces on which several locally most central points exist without there being a simple geometric definition of the corresponding failure modes such as is the case for collapse mechanisms in rigid plastic hinge models for frame structures. Taking as simplified idealized model a model of similarity with the elaborate model but with clearly defined failure modes, the MCFM can be started from each idealized single mode limit state in turn to identify a locally most central point on the elaborate limit state surface. Typically this procedure leads to a fewer number of locally most central failure points on the elaborate limit state surface than existing in the idealized model.

1 INTRODUCTION

To be practicable, reliability analysis of a structure with respect to an implicitly given elaborate limit state often requires a replacement of the limit state surface by an approximating explicitly given surface. In the literature on the topic it is frequently recommended that a quadratic polynomial in the input variables is chosen to define the approximation. The coefficients of the polynomial are then determined by a least square fit to a sample of points of the limit state surface. Choosing the size of this sample somewhat larger than the number of unknown coefficients in the polynomial allows a statistical evaluation of the approximation error within the chosen domain of approximation. The approximation error may then be taken into account in the reliability analysis. This limit state replacement method has been called the Response Surface Method (Veneziano, Casciati, & Faravelli 1983). However, the application of the RSM faces the problem that it is often not known where about in the space of input variables the most central part of the limit state surface is situated. Therefore an iterative procedure is used. The most central point P_i (design point) on the response surface $\mathcal{S}_i$ is found by a suitable search procedure. A new response surface $\mathcal{S}_{i+1}$ is next determined by use of points sampled in some neighborhood of P_i and the design point P_{i+1} on $\mathcal{S}_{i+1}$ is determined. Repetitions are made until no essential change is observed. Quite often a lack of convergence is encountered.

In stead of replacing the complicated limit state surface by some arbitrary mathematical surface one may choose to simplify the mechanical model down to an operable level under preservation of the most dominant geometric, static and physical properties of importance for the definition of the failure event. The principle is then that the design point on the simple limit state surface is determined, whereupon the complicated model is called at this point to determine a correction factor to the resistance variables of the simple model. The correction is based on requiring coincidence of the simple surface with the complicated surface at the design point. If coinciding tangent planes happen to be obtained by this affinity transformation, the two surfaces have the same geometric reliability index. This is checked by an iteration calculation procedure in which the model correction factor is approximated by an inhomogeneous linear function of the input variables. In case of moderate deviations from coinciding tangent planes the approximation can alternatively be checked and corrected by a special method of directional simulation (Ditlevsen 1996). This mechanically based RSM is called the Model Correction Factor Method.

The rigid plastic yield hinge models have turned out to be quite effective geometric response surface models for frame and truss structures with ductile failure behavior (Ditlevsen & Arnbjerg-Nielsen 1994, Johannesen 1997), for models of damaged concrete slaps (Karlsson, Johannesen, & Ditlevsen 1993), and also for large ductile box girder structures (Friis-Hansen 1994, Johannesen & Ditlevsen 1995). The yield hinge model applies as an operable simplification even for stability failure limit states of geometric non-linearity type (Johannesen & Ditlevsen 1993). For stability problems the key issue of the method is to construct the simple yield hinge model such that the set of possible polygonal mechanism geometries include shapes that have some crude similarity with the real displacement shapes that develop when approaching the failure situation.

The experienced success of the MCFM for component reliability analysis invites to study its possibilities as a tool for system reliability analysis. Application of the non-mechanically based response surface method for system reliability purposes requires the determination of the response surface for each of the dominant failure modes. However, for the complicated limit state surface it may not be clear how to classify different regions as different failure modes. Thus the identification of the different regions of needed response surface approximation may be a major problem. On the other hand, when simplifying the system to be governed by a rigid plastic (or a linear-elastic-ideal-plastic) hinge model, a set of distinct mechanisms can be identified. The MCFM applied to each of these collapse mechanisms leads to a corrected set of linear safety margins that may be subject to ordinary series system reliability analysis.

To be self-contained, the paper first recapitulates the general theory of the MCFM following the presentation in (Ditlevsen & Madsen 1995), pp.139-145, as well as the method of uniform directional simulation on a cone.

2 MODEL CORRECTION FACTOR METHOD

For a given structure let $\mathbf{x}_F$ be the vector of all basic variables with physical units that contain the unit of force and let $\mathbf{x}_D$ be the vector of all remaining basic variables (of the type as geometric and dimensionless basic variables). With sufficient generality we can assume that the basic variables are defined such that the units of the elements of $\mathbf{x}_F$ are all proportional to the force unit. Let $\mathbf{x}_F = (\mathbf{x}_S, \mathbf{x}_R)$ be split into the vector $\mathbf{x}_S$ of load variables and the vector $\mathbf{x}_R$ of strength variables, respectively, and consider two limit-state equations between the vectors $\mathbf{x}_S$, $\mathbf{x}_R$, and $\mathbf{x}_D$ to be one defined by an elaborate model, and the other by a simple model formulated as an idealization of the elaborate model.

The input values are assumed to be specified as random vectors $(\mathbf{X}_S, \mathbf{X}_R, \mathbf{X}_D)$ with a given joint probability distribution. The quantity of interest is the probability that a realization of $(\mathbf{X}_S, \mathbf{X}_R, \mathbf{X}_D)$ is obtained in the failure set $\mathcal{F}_r$ of the elaborate (r for "realistic") model. The problem at hand is that the calculation of this probability $P(\mathcal{F}_r)$ is elaborate. Therefore it is attractive to try to take advantage of the simple model by which the probability $P(\mathcal{F}_i)$ of getting a realization in the idealized failure set $\mathcal{F}_i$ can be calculated with less effort than required for the calculation of $P(\mathcal{F}_r)$.

The elaborate limit state and the simple limit state are defined as the set of zero points of the functions $g_r(\mathbf{x}_S, \mathbf{x}_R, \mathbf{x}_D)$ and $g_i(\mathbf{x}_S, \mathbf{x}_R, \mathbf{x}_D)$, respectively, where the first function is suitably regular but not necessarily given in explicit form, and the second function is a less elaborate function than the first. It is assumed that both the safe sets are star-shaped in terms of $\mathbf{x}_S$ with respect to the origin of $\mathbf{x}_S$, that is, for any $(\mathbf{x}_S, \mathbf{x}_R, \mathbf{x}_D)$ each of the two equations

$$g_r(\kappa_r \mathbf{x}_S, \mathbf{x}_R, \mathbf{x}_D)=0 \text{ and } g_i(\kappa_i \mathbf{x}_S, \mathbf{x}_R, \mathbf{x}_D)=0 \qquad (1)$$

has a unique solution with respect to κ_r and κ_i, respectively. Defining the so-called effectivity factor as

$$\nu(\mathbf{x}) = \frac{\kappa_r(\mathbf{x})}{\kappa_i(\mathbf{x})} \qquad (2)$$

where $\mathbf{x} = (\mathbf{x}_S, \mathbf{x}_R, \mathbf{x}_D)$, it is then a consequence of the physical property of dimension homogeneity of the limit-state equations that the limit states defined by each of the equations

$$g_r(\mathbf{x}_S, \mathbf{x}_R, \mathbf{x}_D)=0 \text{ and } g_i(\mathbf{x}_S, \nu(\mathbf{x})\mathbf{x}_R, \mathbf{x}_D)=0 \qquad (3)$$

are identical, see proof in (Ditlevsen & Madsen 1995).

Being identical, the two equations (3) are equally elaborate, of course. However, it is reasonable to expect that the effectivity factor locally can be approximated by a constant. Then the equation $g_i(\mathbf{x}_S, \nu(\mathbf{x})\mathbf{x}_R, \mathbf{x}_D) = 0$ may as an approximation be replaced by the equation $g_i(\mathbf{x}_S, \nu^*\mathbf{x}_R, \mathbf{x}_D) = 0$ applicable in a more or less wide neighborhood of any point $\mathbf{x}^*$ at which $\nu^* = \nu(\mathbf{x}^*)$ is calculated. Thus ν^* acts as a model-correction factor applied to the strength variables in the idealized model. For reliability analysis purposes the best choice of the value of this factor obviously is the one that is obtained at the most central point of the elaborate limit-state surface represented in the standard Gaussian space. Given that $\nu(\mathbf{x})$ actually has a point of stationarity at $\mathbf{x}^*$, that is, given that all the partial derivatives of $\nu(\mathbf{x})$ are zero at $\mathbf{x}^*$, then the two limit-state surfaces defined by

$g_r(\mathbf{x}_S, \mathbf{x}_R, \mathbf{x}_D) = 0$ and $g_i(\mathbf{x}_S, \nu^* \mathbf{x}_R, \mathbf{x}_D) = 0$ are tangential to each other at $\mathbf{x}^*$. Thus the two limit-state surfaces have $\mathbf{x}^*$ in common as a point that satisfies the necessary conditions for being a most central point also for $g_i(\mathbf{x}_S, \nu^* \mathbf{x}_R, \mathbf{x}_D) = 0$.

The search for the point $\mathbf{x}^*$ with these properties may start by applying FORM to $g_i(\mathbf{x}_S, \nu^* \mathbf{x}_R, \mathbf{x}_D) = 0$ with ν^* put to some judgementally chosen value, $\nu^* = 1$, say. This determines a first approximation to the most central point. At this point the equation $g_r(\kappa_r \mathbf{x}_S, \mathbf{x}_R, \mathbf{x}_D) = 0$ is solved with respect to κ_r, the solution being a new value of the correction factor. Using this factor in $g_i(\mathbf{x}_S, \nu^* \mathbf{x}_R, \mathbf{x}_D) = 0$ a new FORM analysis gives a new approximation to the most central limit state point. At this new point, $g_r(\kappa_r \mathbf{x}_S, \mathbf{x}_R, \mathbf{x}_D) = 0$ is again solved with respect to κ_r, and the ratio between this solution and the previous solution is a new assessment of the correction factor. If convergence to a fixed correction factor value ν is obtained by iterative application of this calculation, then the limit point $\mathbf{x}$ is a solution to both $g_r(\mathbf{x}_S, \mathbf{x}_R, \mathbf{x}_D) = 0$ and $g_i(\mathbf{x}_S, \nu^* \mathbf{x}_R, \mathbf{x}_D) = 0$. However, it is not sure that $\mathbf{x}$ is a point of coinciding tangent planes between the surface defined by $g_i(\mathbf{x}_S, \nu^* \mathbf{x}_R, \mathbf{x}_D) = 0$ (on which it is a locally most central point, per definition) and the surface defined by $g_r(\mathbf{x}_S, \mathbf{x}_R, \mathbf{x}_D) = 0$. On the other hand, by an intelligent on physical principles based modeling of the idealized limit-state surface it is intended, of course, that it gets reasonable similarity with the elaborate limit-state surface within the domain of importance. Therefore it should be expected that the deviation from coinciding tangent planes is small in many cases. In fact, several examples have shown that even before the first iteration quite good estimates of the geometric reliability index (Hasofer-Lind) can be obtained.

A check and possible correction for not having coinciding tangent planes after convergence of this zero order MCFM can be made by replacing the effectivity factor $\nu(\mathbf{x})$ by its first order Taylor expansion $\tilde{\nu}(\mathbf{x})$ at the limit point $\mathbf{x}$. If necessary, improvements can be made by proceeding with first order MCFM, which simply is to determine the most central point on $g_i(\mathbf{x}_S, \tilde{\nu}(\mathbf{x})\mathbf{x}_R, \mathbf{x}_D) = 0$ and thereupon go on with a new first order Taylor expansion of $\nu(\mathbf{x})$ etc. iteratively until stop according to a suitable criterion. The first order Taylor expansion of $\nu(\mathbf{x})$ is determined by the normal vector $\mathbf{n}$ to the elaborate limit-state surface at $\mathbf{x}$. To calculate the normal vector numerically takes as many calculations of points on the elaborate limit-state surface in the vicinity of $\mathbf{x}$ as the dimension of the space. For verifying that there is coinciding tangent planes or making correction for a moderate deviation from coinciding tangent planes it can therefore be advantageous as a practicable alternative to apply directional simulation on a cone (Ditlevsen 1996). Cone simulation is shortly recapitulated in Section 4.

3 LINEAR ELASTIC-RIGID SOFTENING FRAME EXAMPLES

The model correction factor method system reliability analysis is in the following applied to frame structures of shape as portal frames put on top of each other. As the bending moment in any beam cross-section of the frame increased from zero in absolute value, the moment-curvature constitutive relation is linear elastic until a random value of the bending moment is reached. Above this value of the bending moment the beam cross-section has a softening elastic behavior. Since the local maximal bending moments are at the two supports, at the frame corners, and at the points of single force application (assuming that there is no distributed load on the frame element), a model with possible softening behavior solely at these positions is applicable as an approximation for carrying capacity analysis. Thus a hinge model is introduced with potential hinges placed as indicated in Fig. 1. The bending moment-angular rotation relation of a hinge is assumed to have the form

$$\frac{M}{M_0} = \text{sgn}(\theta)\frac{2(1+|\theta|/\theta_0)}{1+(1+|\theta|/\theta_0)^2} \tag{4}$$

where the strength level M_0 is randomly varying from hinge to hinge and θ_0 is a softening parameter assumed to be non-random and the same for all hinges, Fig. 1. It is noted that as long as θ increases, then there is no difference between this elastic constitutive model and a softening plasticity model. However, if θ turns from increasing to decreasing then the bending moment turns from decreasing to increasing until $\theta = 0$ is reached, a per definition impossible behavior for a plastic material. Even though proportional loading without decrease of the load level is applied, it is not sure that decrease of the angular rotations in some

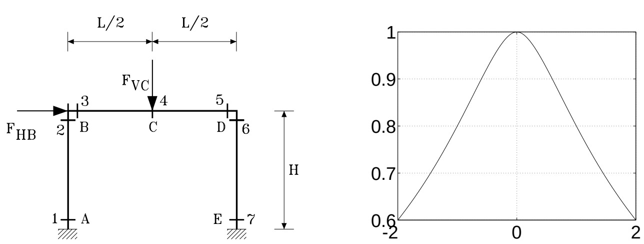

Figure 1. Left: Portal frame with load configuration and corresponding potential hinges 1 to 7. Right: Rigid softening-elastic rotation-moment relation (4) for the beam cross-sections ($x = \theta/\theta_0, y = M/M_0$).

hinges will not take place before the carrying capacity of the frame is reached. However, the finite element calculations show that almost all of the realizations of the frame have the carrying capacity reached at the first local maximum of the load. Presumably therefore, the effect is minor of the less realistic elasticity assumption relative to a more realistic plasticity assumption. For simplicity of definition the carrying capacity is for a given load configuration (i.e. for given ratio between vertical and horizontal load) in the present investigation defined to be the first reached maximum of the load.

3.1 *Portal frame*

The first series of examples of MCFM-analyses are for the portal frame. These examples are the same as considered in (Ditlevsen & Arnbjerg-Nielsen 1994) for illustrating simple first order MCFM and in (Johannesen 1997) for illustrating the improvement obtained by applying iterated first order MCFM.

The values of M_0 in the 7 potential hinges are taken to be outcomes of a jointly Gaussian random vector with a common mean value 0.1 MN in hinges 1, 2, 6, and 7 of the frame legs, and 0.2 MN in hinges 3, 4, and 5 of the traverse. For all hinges, the coefficient of variation of M_0 is 0.1. The correlation coefficients between the M_0-variables within each leg separately and within the traverse have the same value 0.9. All other correlation coefficients are zero. The geometry of the frame is given by L =5.0 m, and $H = 3.0$ m, see Fig. 1. The beam elements have Young's modulus $E = 2.1 \times 10^5$ MN/m^2 and moment of inertia $I = 36.9 \times 10^{-6}$ m^4.

The load level is defined by the load scaling factor λ applied to the load reference value $F_0 = 0.033$ MN. The horizontal load is $F_{HB} = \lambda F_0$ and the vertical load is $F_{VC} = \lambda\eta F_0$, where η is a constant. All combinations of the parameter values $\eta = 0.394$, 0.667, and $\theta_0 = 0.02$, 0.04, 0.06 have been analyzed.

For each of the parameter combinations repeated simulation of realizations of the random variables of the problem gives an empirical distribution function of the carrying capacity of the frame in terms of the first local maximum of λ as λ is increased stepwise from zero. This calculation may be made as a stepwise finite element calculation giving the carrying capacity as the value of λ for which there is divergence of a modified Newton-Raphson iteration scheme. This divergence corresponds to the first occurrence of a singular tangent stiffness matrix or, alternatively, to the occurrence of the first peak on the load parameter displacement curve (defined for some suitably chosen displacement measure). Alternative solution procedures (arch-length control, displacement control) with the associated phenomenon of several load parameter peaks are considered in (Arnbjerg-Nielsen 1991) and (Johannesen 1997).

Clearly the simulation procedure leads directly to a system carrying capacity distribution function. The possibility of saving large computational efforts makes it of considerable interest to investigate whether it is possible to consider the failure set as composed by simple individual failure modes for which approximate failure probabilities may be calculated by fast methods such as the standard first or second order reliability method (FORM or SORM), and such that well known series system methods can be used to obtain the system reliability. However, for the considered example there are no clear mechanism geometries such as it is the case for rigid ideal plastic hinge models.

The MCFM points at a solution to this problem. In fact, as the simplified model one may reasonably choose the corresponding rigid ideal plastic hinge model. Then there are 10 safety margins $G_i, i \in \{1, 2, ..., 10\}$, with failure domains $G_i < 0$, defined as the difference between the plastic dissipation of energy and the work done by the loads during mechanism movement:

$$\begin{bmatrix} G_1 \\ G_2 \\ G_3 \\ G_4 \\ G_5 \\ G_6 \\ G_7 \\ G_8 \\ G_9 \\ G_{10} \end{bmatrix} = \begin{bmatrix} 1&0&0&2&0&2&1 \\ 1&0&0&2&2&0&1 \\ 0&1&0&2&0&1&0 \\ 0&1&0&2&1&0&0 \\ 0&0&1&2&0&1&0 \\ 1&0&1&2&1&0&0 \\ 1&1&0&0&0&1&1 \\ 1&1&0&0&1&0&1 \\ 1&0&1&0&0&1&1 \\ 1&0&1&0&1&0&1 \end{bmatrix} \begin{bmatrix} M_{01} \\ M_{02} \\ M_{03} \\ M_{04} \\ M_{05} \\ M_{06} \\ M_{07} \end{bmatrix} - \begin{bmatrix} 1&1 \\ 1&1 \\ 0&1 \\ 0&1 \\ 0&1 \\ 0&1 \\ 1&0 \\ 1&0 \\ 1&0 \\ 1&0 \end{bmatrix} \begin{bmatrix} HF_{HB} \\ \dfrac{LF_{VC}}{2} \end{bmatrix} \quad (5)$$

where the the forces F_{HB} and F_{VC} are defined in Fig. 1 and M_{01}, M_{02},...,M_{07} are the yield moments in the 7 hinges. In (5), G_1 and G_2 represent combined beam and sway mechanisms, G_3, G_4, G_5, and G_6 represent beam mechanisms, and G_7, G_8, G_9, and G_{10} represent sway mechanisms.

The MCFM analysis is hereafter carried out as follows: i) For each of the 10 rigid plastic collapse modes the most central point is determined together with the corresponding reliability index. ii) For each of these points a zero order MCFM analysis is made giving a first approximation to a set of locally most central points on the limit state surface for the frame structure with softening hinges. iii) First order MCFM analyses are made starting at each of the non-coinciding points obtained in ii. iv) Finally only considering the non-coinciding tangent hyperplanes corresponding to the locally most central points obtained in iii, a series system reliability analysis is made by assessing lower and upper bounds, say.

As expected, several of the MCFM analyses for the individual rigid plastic mechanisms converge towards a common locally most central point. In order to account for limited accuracy in the optimization algorithm in the reliability evaluation and the FE-algorithm in the g_r-function evaluation, limit states for which the locally most central points are equal within a pre-specified tolerance are represented by a single linearized limit state. The criterion is set to $(\mathbf{u}_i^{*\mathsf{T}}\mathbf{u}_j^*)/(\|\mathbf{u}_i^*\|\|\mathbf{u}_j^*\|) > 0.99$ where $\mathbf{u}$ is the vector of input variables in standard Gaussian space and * and subscripts refer to local most central point and rigid plastic collapse mode numbers, respectively.

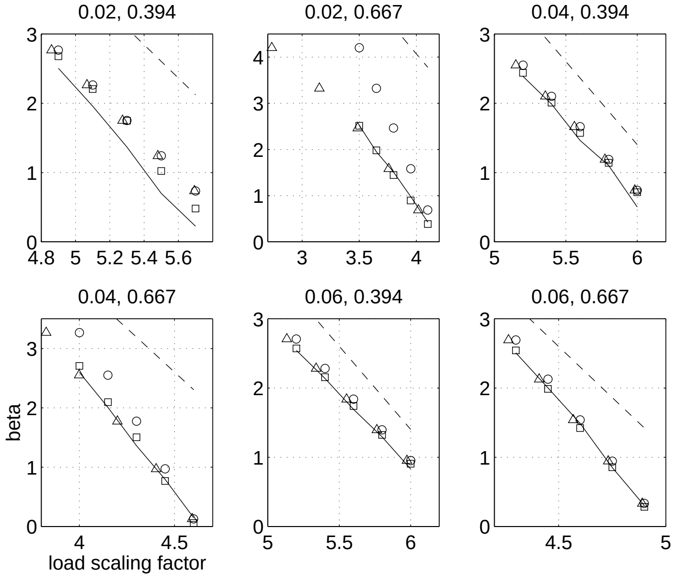

Figure 2. Smallest single failure mode reliability indices β from MCFM computations and estimated system reliability index obtained by directional simulation as function of the load scaling factor λ for two deterministic load levels $\eta = 0.394, 0.667$, and softening parameter $\theta_0 = 0.02$, 0.04, 0.06. Dashed curve: non-corrected most probable rigid plastic mechanism. Circles: converged zero order MCFM. Triangles: first iteration first order MCFM. Squares: converged first order MCFM. Full curve: directional simulation estimate of system reliability.

3.2 *Results for portal frame*

The smallest single failure mode reliability indices obtained by MCFM analysis are shown as functions of the load scaling factor λ in Fig. 2, reproduced from (Johannesen 1997). The fully drawn curves show the system reliability indices as obtained by uniform directional simulation. The curves correspond to the average of the upper and lower 90% confidence interval bounds obtained for sample sizes for which the difference between the bounds is about 0.3 on the β-scale (assuming asymptotic normality of the failure probability estimator). Directional simulations have been carried out until stop for a difference between the confidence limits of about 0.05 after publishing (Johannesen 1997) . The corresponding sample sizes ranged roughly from 200 to 7000, increasing with the reliability index. However, only a very small change of the average was observed compared to the curves in Fig. 2.

The system reliability bounds are very narrow in all investigated parameter cases. In fact, the 10 different initial points corresponding to each of the 10 rigid plastic mechanisms of the idealized model converged into no more than 3 different locally most central points on the limit state surface for the softening frame in 59 out of 60 examples. The bounds are not shown in the diagrams because they are found all to be closely coinciding with the simulated system results.

For larger problems, computation efficiency improvements are relevant. Straightforward means for reducing the efforts are to consider only a set of most probable idealized mechanisms as starts for zero order MCFM computations, to abort a MCFM computation before converged if the obtained point sequence tends to converge to a point in the vicinity of a point obtained in a MCFM analysis for a previously used mechanism, and to abort a MCFM computation before converged if the reliability level stabilizes at a level much higher than the reliability level obtained at any of the previous MCFM analyses.

4 SAMPLING ON A CONE

Consider a spherical cone with vertex at the origin of the n-dimensional real space and with axis in the direction of the unit vector $\mathbf{e}_1$ of the first coordinate axis. Let the angle between $\mathbf{e}_1$ and any generatrix of the cone be γ. Moreover, let $\mathbf{X} = (X_1, \ldots, X_n)$ be an n-dimensional standard Gaussian random vector. Then the random directional unit vector $\mathbf{A}$ defined by

$$\mathbf{A}\|\mathbf{Z}\| = \mathbf{Z} = \mathbf{e}_1 + \frac{\mathbf{X} - X_1\mathbf{e}_1}{\|\mathbf{X} - X_1\mathbf{e}_1\|}\tan\gamma \qquad (6)$$

has a uniform distribution on the intersection between the cone and the unit sphere. The formula (6) makes it quite simple to simulate realizations of $\mathbf{A}$. Moreover, by simple rotation of the coordinate system the axis of the cone can be directed in any specified direction of the space. Therefore it is sufficient in the following to let the axis direction be coincident with the unit vector $\mathbf{e}_1$.

Let $\mathcal{F} \subset \mathbb{R}^n$ be a set with a boundary $\partial\mathcal{F}$ that is cut at most at a single point by any generatrix of the cone in direction $\mathbf{e}_1$. The distance to the cut point from the origin in the random direction $\mathbf{A}$ written as $r(\mathbf{A})$ is obtained by solving the equation $g[r(\mathbf{A})\mathbf{A}] = 0$, where $g(\mathbf{x}) = 0$ is the equation of the boundary surface $\partial\mathcal{F}$. If, in particular, the surface $\partial\mathcal{F}$ is a hyperplane orthogonal to $\mathbf{e}_1$ at the distance d from the origin, then all values of the sampling variable

$$S = \frac{1}{d} r(\mathbf{A}) \cos\gamma \qquad (7)$$

are 1. However, if $\partial\mathcal{F}$ is a hyperplane not orthogonal to $\mathbf{e}_1$ through the point $\mathbf{e}_1 d$, then S will show random deviation from 1. In fact, the distribution of S gives information about the normal vector to $\partial\mathcal{F}$ at the point $\mathbf{e}_1 d$ given that this point is a point of $\partial\mathcal{F}$ and that $\partial\mathcal{F}$ is flat at this point (in the sense that FORM and SORM give approximately the same generalized reliability index if the normal vector to $\partial\mathcal{F}$ at the point should happen to be coincident with $\mathbf{e}_1$). Assume that this flatness condition allows to replace the surface $\partial\mathcal{F}$ with a hyperplane that has the normal unit vector $\mathbf{n}$ at an angle ω with $\mathbf{e}_1$. Moreover, let γ be chosen such that $\gamma \ll \frac{\pi}{2} - \omega$. As noted, S is the constant 1 if and only if $\omega = 0$.

It is shown in (Ditlevsen 1996) that the density function of the sampling variable S is given by

$$f_S(s) \propto \frac{1}{s^2 \tan\omega}\left[1 - \left(\frac{1-s}{s\tan\gamma\tan\omega}\right)^2\right]^{\frac{n-4}{2}} \qquad (8)$$

for $1 - \tan\gamma\tan\omega \leq s^{-1} \leq 1 + \tan\gamma\tan\omega$, and zero otherwise. Let $s_1, \ldots, s_m$ be a sample of S generated from a sample $\mathbf{z_1}, \ldots, \mathbf{z}_m$ of the random vector $\mathbf{Z}$ in (6). The log-likelihood function for ω then becomes

$$L[\omega \mid s_1, \ldots, s_m] = \sum_{i=1}^{m} \log[f_S(s_i \mid \omega)] \qquad (9)$$

defined for $0 \leq \omega < \frac{\pi}{2} - \gamma$. A point estimate of ω is the point of maximal log-likelihood.

The applicability of this method of estimating the angular deviation of the normal vector $\mathbf{n}$ from the first axis unit vector $\mathbf{e}_1$ has been tested by example calculations. Obviously the cone simulation method is particularly advantageous in cases where the number of calculations of $r(\mathbf{A})$ is considerably less than that needed for deterministic numerical calculation of the normal vector, that is, considerably less than the dimension n of the space. It is demonstrated in (Ditlevsen 1996) that rather small sample sizes as compared to the dimension of the considered space are sufficient to obtain reasonably accurate estimates. Test examples with $\tan\gamma = 0.05$, 0.1 and 0.2 show that the value of the cone angle γ has negligible influence on the maximum likelihood estimates of ω.

Within the range of small values of ω the cone simulation method is anticipated to be effective simply because the sampling variable S becomes the constant 1 for $\omega = 0$ and a plane surface $\partial\mathcal{F}$. This is the asymptotic verification situation often met when applying response surface methods for reliability analysis with elaborate limit-state surfaces in high dimensions.

To check whether or not there is coinciding tangent planes or only minor deviation from coinciding tangent planes after a 0th order MCFM analysis it can therefore be advantageous as a practicable alternative to 1st order MCFM analysis to apply directional simulation on a cone. Given that the elaborate limit-state surface is almost flat at the most central limit-state point and the angle between the two limit-state surfaces is small, an estimate of the geometric reliability index is

$$\beta = \|\mathbf{x}\| \cos\hat{\omega} \qquad (10)$$

where $\hat{\omega}$ is the estimate of the angle ω. By making simple test simulations the smallest needed sample size that is likely to provide sufficient accuracy in a given application can be estimated before the start of time consuming calculations of points on the elaborate limit-state surface.

Relying on the similarity between the two surfaces, the flatness can be judged by comparing the reliability results obtained by applying both FORM (curvature independent results) and SORM (curvature dependent results) to the idealized limit-state surface. Another possibility is to test whether there are significant deviations of the simulated sample from the distribution (8) obtained from the hyperplane assumption.

5 FRAMES WITH 2 AND 8 TRAVERSES

The second series of examples is for a 2-story frame constructed by placing two copies of the portal frame of the previous example on top of each other. This frame structure gets 14 potential hinges with the constitutive behavior defined by (4). The maximal hinge moments M_0 have coefficient of variation 0.1, and they are taken to be uncorrelated. The mean value is 0.1 MNm and 0.2 MNm in columns and traverses, respectively. The softening parameter θ_0 is 0.05 in all hinges.

The horizontal loads are applied at the 2 traverse-column joints and the vertical loads are applied at the centers of the 2 traverses. All loads are independent Gaussian random variables with coefficient of variation 0.1. Two different load configurations are considered: 1) mean vertical loads = 0.1 MN, mean horizontal loads each varied from 0.03 to 0.06 MN in steps of 0.01 MN. 2) all four loads of common mean value varied from 0.04 to 0.055 MN in steps of 0.005 MN. The rigid-plastic limit-state model is chosen to be made up of 12 mechanisms. Mechanisms 1 and 2 are beam mechanisms, 3 to 6 are sway and combined beam/sway

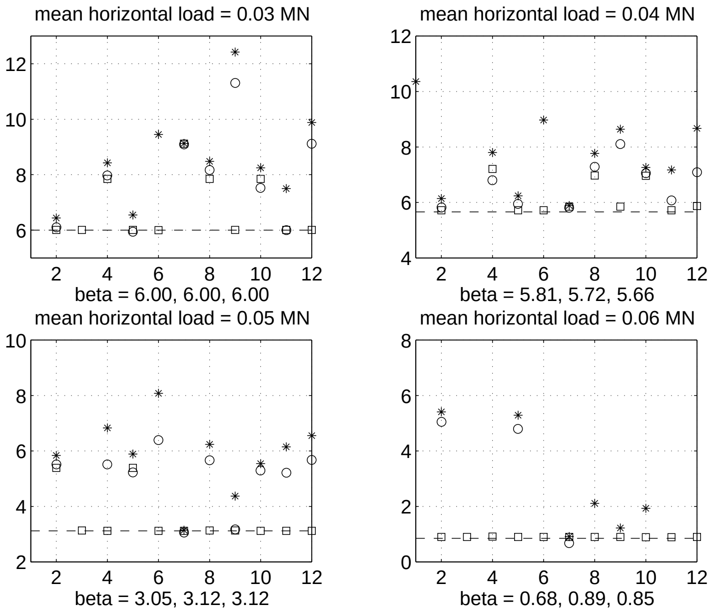

Figure 3. Collapse mode reliability index approximations obtained by 0th order MCFM (star), cone simulation corrected 0th order MCFM (circle), 1st order MCFM (square) for each of 12 start mechanisms, and resulting system reliability index (dashed line). The listed beta values correspond to smallest circle point, smallest square point, and system bounds. (Two-story frame, same vertical mean load = 0.1 MN per traverse in all cases).

mechanism involving the top traverse, 7 and 8 are sway and combined beam/sway mechanisms involving the bottom traverse, and 9 to 12 are sway and combined sway/beam mechanisms involving both traverses.

Zero order and first order MCFM analyses are made with start from each of these mechanisms. Cone simulations are made for obtaining correction factors to the zero order reliability indices according to (10). Both the zero order MCFM reliability indices and their corrected values together with the first order MCFM reliability indices are plotted in Figs. 3 and 4 for each starting mechanism numbered 1 to 12. It is seen that start from several different mechanisms leads to the same first order reliability index. In fact, the corresponding locally most central failure points coincide within tolerance settings. This observation shows that the softening frame structure has a considerably smaller number of different failure modes than the idealized rigid plastic frame structure. The system reliability index (dashed line in the figures) is calculated from the corresponding reduced set of linear safety margins using well-known bounding technique. In this example the upper and lower bounds are practically coinciding and also almost coinciding with the smallest of the first order MCFM reliability indices. Both pure beam mechanisms and beam/sway mechanisms appear as most probable mechanisms. Between 1 and 4 single failure modes entered the system reliability analysis. Less restrictive reliability requirements for pure beam mechanisms than for sway mechanisms may imply that pure beam mechanisms possibly should be considered separately.

It is observed that the corrected zero order reliability indices (circles) are mostly larger than the first order reliability indices (squares) but, interestingly, also that in all cases the smallest corrected zero order reliability index is practically coinciding with the smallest first order reliability index.

The sample size used in the cone simulations for correcting the zero order MCFM reliability indices is chosen to $m = 14 \approx 0.8\,n$, where $n = 18$ is the number of random variables in the problem. For this sample size the computational savings compared to direct gradient calculations are not substantial. However, for problems with a considerably larger number n of random variables than in this test example a much lower fraction of n is sufficient, e.g. $0.2n$ (Ditlevsen 1996).

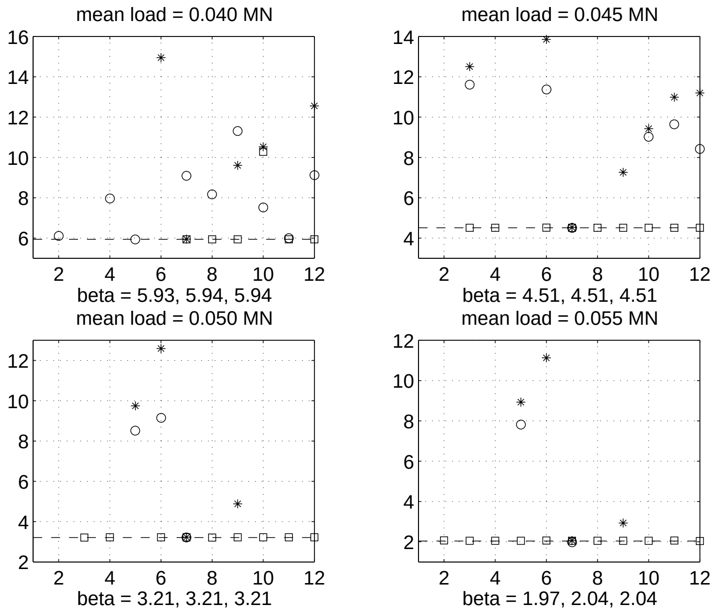

Figure 4. Reliability index results for two-story frame with same mean value for all loads. Legend otherwise as in Fig. 3.

Finally an 8-story frame constructed by placing 8 copies of the portal frame in Fig. 1 on top of each other has been analyzed by MCFM. This frame structure gets 56 potential hinges with the constitutive behavior defined by (4) and quantitative properties as for the two-story frame. The horizontal loads are applied at the 8 traverse-column joints and the vertical loads are applied at the centers of the 8 traverses. All loads are independent Gaussian random variables with coefficient of variation 0.1. All 16 loads have common mean value with two cases 0.040 and 0.048 MN considered. Thus the total number of random variables is $n = 72$.

The results are plotted in Fig. 5 using 24 representative mechanisms for the corresponding idealized rigid plastic frame. The same conclusions as for the 2-story frame example can be drawn in this example. The 24 mechanisms end up to give 8 and 12 failure modes for the mean load cases 0.40 and 0.48 MN, respectively. In the load case 0.48 MN, the rather small upper and lower system reliability index bounds deviate somewhat from each other (both plotted by dashed lines).

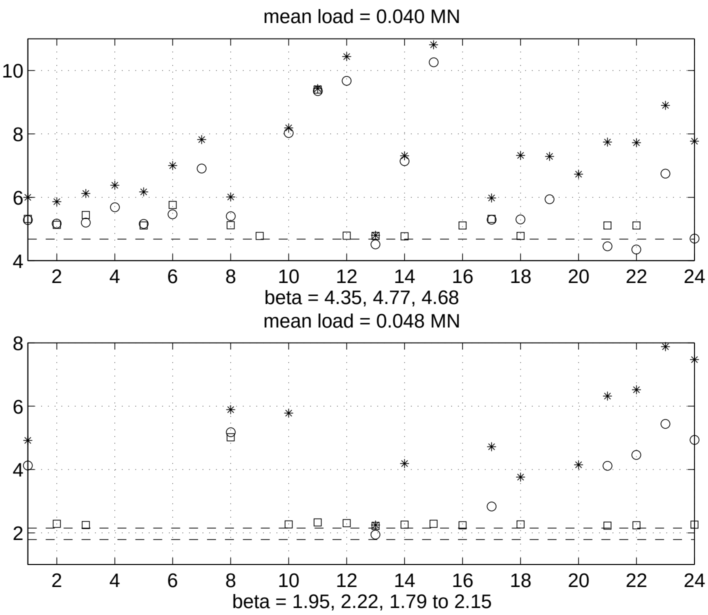

Figure 5. Reliability index results for eight-story frame with same mean value for all loads. Legend otherwise as in Fig. 3.

Comments on convergence

Some result points are missing for different mechanisms in Figs 3, 4, and 5 corresponding to cases where an analysis has failed or has not been carried out due to failure of preceding analyses that are required as input. For the MCFM, break down of analyses occurs due to divergence of iterations. An angle simulation fails if a sample occurs for which no root $r(\mathbf{A}\ )$ smaller than a pre-specified upper limit is found. In such cases the simulation is aborted. Moreover, if the final estimated gradient angle is greater than a pre-specified upper limit ω_{max}, the simulation is considered failed. For the 2- and 8-story frames, $\tan\omega_{max}$ is set to 1 and 2, respectively. The cone angle γ is set to $\gamma = \arctan(0.1)$.

It is noted that in case the 0th order MCFM analysis fails for some mechanism, then first order MCFM analysis may still be carried out because convergence is obtained in most cases irrespective of a diverged starting point from the 0th order MCFM.

6 CONCLUSIONS

For the frame structures herein it is seen that the complete set of first order MCFM analyses identifies a decreased set of different local failure modes. These contribute to a total system reliability index by well-known bounding methods. The system effect is observed to be small in these examples and mostly the system reliability index is only slightly smaller than the smallest of the single mode reliability indices. A useful observation is that if the computationally much faster zero order MCFM is applied combined with reliability index correction by use of directional simulation on a cone, then the smallest of these corrected reliability indices are in most considered cases approximately equal to the smallest of the first order MCFM reliability indices. This is particularly useful observation when facing reliability problems with a large number of random variables where fully converged first order MCFM (or any other response surface method) may require extensive computer time. The failure modes identified in this way may subsequently be subject to further analysis.

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