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Measuring uncertainty correction by pairing

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ABSTRACT: Under certain circumstances a pairing of measurements from two independent and possibly inaccurate measuring methods reveals information about the population of measuring objects as well as about the two measuring error populations. This pairing method in combination with stochastic interpolation principles is the basis for a method of getting information about the uncertainty related to measurements of spatial variations of soil properties and of separating the uncertainty population and the property population from each other. The method is developed for the following particular application. Elasto–plastic continuum mechanics predicts that there is almost proportionality between the undrained shear strength c_v as measured by the vane test and the cone tip resistance q_c, both imagined to be measured at the same point of an ideal saturated clay. Taking this as a "law", observed deviations of the measured pairs (c_v,q_c) from being situated on the same straight line through the origin in the (c_v,q_c) coordinate system must be attributed to measuring uncertainty. In fact, large deviations are observed in practice because in the pair (c_v,q_c) assigned to a given point the value of q_c, say, must be obtained by interpolation between values of q_c measured at other points of the soil body. Acceptance of the proportionality law makes it possible by the pairing method to estimate the proportionality constant as well as the measuring uncertainty of both c_v and q_c in terms of probability distributions. This leads to the transformation of the random field of "measured" q_c values into the random field of "true" undrained shear strength values. The developed technique has been successfully applied to an extensive set of filtered CPT (cone penetration test) cone tip resistance measurements made in the Storebælt clay till in Denmark. This investigation has served the preparation of the anchor block design for the suspension bridge presently under construction.

Introduction

The problem treated in this paper is a special case of the following general measuring uncertainty evaluation problem: A sample is drawn from some unknown population Ω (object population) and each element of the sample is characterized by a measured value. The measurement procedure is assumed to be less than perfect. On each measured value it introduces an error drawn from some unknown population M_1. Without knowing anything about population M_1 it is clearly not possible on the basis of the obtained sample of values to infer anything about the properties of population Ω. However, the situation is different if each element of the sample from Ω also is characterized by a measured value obtained by use of another independent measuring method with error population M_2. It is shown in the next section that if both measuring methods besides being independent are such that the two mean errors for a given object are independent of the error–free value that should be assigned to the object then it is possible to estimate the variances of each of the three value populations corresponding to Ω, M_1, M_2 on the basis of the sample of pairs of measured values. In order to obtain estimates of the mean values of the three value populations it is necessary to assume that at least one of the measuring methods deliver unbiased measurements, that is, that the mean error is zero.

The principle of using two inaccurate measuring methods on the same sample of objects from Ω to obtain estimates of population parameters that characterize Ω may be relevant for developing measuring procedures by which large samples of pairs of observations can be obtained at low costs. This may turn out to be particularly relevant in connection with data collection for reliability evaluation of existing structures.

Under certain conditions the principles of the method are even applicable in cases of destructive testing. This is the situation in this paper. Assume that some material property varies in space as a random field. By the process of measuring the property at a point the material is changed irreversibly or even destructed within a certain neighbourhood of the point. Therefore the property cannot be remeasured by some physical measuring device applied at the same point. However, if the first measurement method is applied for one set of points and the second measurement method is applied for another set of points these two sets of measurements can in a certain sense be paired by use of a stochastic interpolation procedure (kriging) based on the random field model of the property variation. At each point of measurement by the first method a measured value is paired with a probability distribution derived by stochastic interpolation between the measured values obtained by the second method. By suitable assumptions about the joint probabilistic structure of the measurement error fields and the "true" property field it is then possible to pull out the "true" field from the measured field. These assumptions are generalizations of the simple independence type assumptions made in the elementary pairing method described in the next section.

Pairing method

Let a random quantity Z be defined on the object population Ω and let it be measured by two independent measuring methods giving the measures X_1 and X_2 respectively. Then the measuring errors are Y_1 and Y_2 and $Z = X_1 + Y_1 = X_2 + Y_2$. The conditional covariance between Y_1, Y_2 given Z is zero, that is, $\mathrm{Cov}[Y_1,Y_2|Z] = 0$, which implies that $\mathrm{Cov}[X_1,X_2|Z] = \mathrm{Cov}[Z-Y_1,Z-Y_2|Z] = 0$.

Under the assumption that the conditional means $E[Y_1|Z]$ and $E[Y_2|Z]$ do not depend on Z, it then follows from the total representation theorem (Ditlevsen (1981), p. 56: $\mathrm{Cov}[X_1,X_2] = E[\mathrm{Cov}[X_1,X_2|Z]] + \mathrm{Cov}[E[X_1|Z],E[X_2|Z]] = E[0] + \mathrm{Cov}[Z,Z]$) that

$$\mathrm{Cov}[X_1,X_2] = \mathrm{Var}[Z] \qquad (1)$$

Thus the variance of the "true" quantity Z can be estimated by estimating the covariance between the results of the two measuring methods. Since $\mathrm{Cov}[X_1,Y_1] = E[\mathrm{Cov}[X_1,Y_1|Z]] = E[\mathrm{Cov}[X_1,Z-X_1|Z] = E[-\mathrm{Var}[X_1|Z]] = -\mathrm{Var}[X_1] + \mathrm{Var}[E[X_1|Z]] = -\mathrm{Var}[X_1] + \mathrm{Var}[E[Z-Y_1|Z]] = -\mathrm{Var}[X_1] + \mathrm{Var}[Z]$ it follows from $\mathrm{Var}[Z] = \mathrm{Var}[X_1] + \mathrm{Var}[Y_1] + 2\mathrm{Cov}[X_1,Y_1]$ that $\mathrm{Var}[Z] = \mathrm{Var}[X_1] - \mathrm{Var}[Y_1]$. Thus the variance of the measuring error Y_1 by use of (1) is obtained as

$$\mathrm{Var}[Y_1] = \mathrm{Var}[X_1] - \mathrm{Cov}[X_1,X_2] \qquad (2)$$

By symmetry, $\mathrm{Var}[Y_2]$ is obtained by interchanging X_1 and X_2.

The following generalizes this idea of pairing the results of two different inaccurate measuring methods and never the less obtain accurate information about the variability of the actual random object. The idea is specifically extended to a situation where the objects are the values of a random field realization. Herein the object population is represented by the "true" field of undrained shear strengths in a saturated clay. Two different inaccurate measuring methods are used at two different sets of points of which the one, S_1, may be more dense than the other, S_2. The pairing is made by stochastic interpolation to the points of S_2 between the measured values at the points of S_1. Rather than being a number, at least one of the elements in the pair becomes a probability distribution obtained by the interpolation. Also there will be stochastic dependence between the different pairs. Herein the two measuring methods are the CPT (cone penetration test) method and the vane test method respectively.

Measurement error modeling

Let $\mathbf{Z} = \mathbf{X} + \mathbf{Y}$ be a vector of logarithms of cone tip resistances obtained by imagined perfect CPT measurements. The vector $\mathbf{X}$ corresponds to the imperfectly measured CPT values while $\mathbf{Y}$ contains the measuring errors. The CPT values refer to the points at which the vane tests are made. The CPT values are therefore not obtained directly but are results of suitable interpolations between physically measured CPT values at other points. Thus the interpolation results are considered as imperfect measurements that can be paired with the physical measurements obtained by the vane test. The logarithms of the vane test measurements are contained in the vector $\mathbf{X}_V$.

Modeling assumption 1a: $E[Y|Z]$ *is independent of* Z.

Since $Cov[Z,Y'|Z] = 0$ (where prime ' means "transpose") it follows by use of this assumption in the total representation theorem (Ditlevsen (1981), p. 90) that $Cov[X+Y,Y'] = Cov[E[Z|Z], E[Y|Z]'] = 0$ implying that

$$Cov[X,Y'] = -Cov[Y,Y'] \tag{3}$$

Substituting this into $Cov[Z,Z'] = Cov[X+Y,(X+Y)'] = Cov[X,X'] + Cov[Y,X'] + Cov[X,Y'] + Cov[Y,Y']$ gives

$$Cov[Z,Z'] = Cov[X,X'] - Cov[Y,Y'] \tag{4}$$

Modeling assumption 2: $Cov[Y,Y']$ *is proportional to* $Cov[X,X']$.

For the homogeneous case of common standard deviation σ of all elements of X we have

$$Cov[X,X'] = \sigma^2 P_X \tag{5}$$

where P_X is the correlation matrix of X. It then follows from (4) and the modeling assumption 2 that

$$Cov[Z,Z'] = \delta^2 P_X, \quad \delta^2 \leq \sigma^2 \tag{6}$$

Remark Let $Z_1,...,Z_n$ and $X_1,...,X_n$ be the elements of Z and X respectively. For the interpretation of the modeling assumption 2 it is interesting to note that it is equivalent to the assumption that the linear regression of Z_i on X depends solely on X_i and that the corresponding regression coefficient is independent of i. In fact, since $\hat{E}[Z|X] = E[Z] + Cov[Z,X'] Cov[X,X']^{-1} (X-E[X])$ this assumption implies that $Cov[Z,X'] Cov[X,X']^{-1} = aI$ for some constant a, where I is the unit matrix. Substituting $Z = X + Y$ and using (3) then give the equation $I - Cov[Y,Y'] Cov[X,X']^{-1} = aI$ from which it follows that $Cov[Y,Y'] = (1-a) Cov[X,X']$. Since the diagonals on both sides must be non-negative it follows that $a \leq 1$. By use of (4) it follows similarly that $a \geq 0$. Setting $a = (\delta/\sigma)^2$ we obtain (6). □

Elasto–plastic continuum mechanics predicts that there is almost proportionality between the undrained shear strength c_v as measured by the vane test and the cone tip resistance q_c, both imagined to be measured at the same point of an ideal saturated clay. Taking this as a "law", there is a constant c such that for all practical purposes we have

$$Z = X_v + Y_v + ce \tag{7}$$

where $e' = [1 \dots 1]$ and where Y_v contains the measuring errors by the vane tests.

Modeling assumption 1b: $E[Y_v|Z]$ *is independent of* Z.

As in the deduction of (4) it follows from (7) that the relations obtained under the modeling assumption 1a hold with X and Y replaced by X_v and Y_v respectively. In particular we have $Cov[Z,Y_v'] = 0$.

Modeling assumption 3: The measuring errors in different vane tests are mutually independent and the logarithms of the measuring errors have common standard deviation γ.

This assumption says that $Cov[Y_v,Y_v'] = \gamma^2 I$ where I is the unit matrix.

Modeling assumption 4: The triple (Z,Y,Y_v) *is jointly Gaussian.*

This assumption implies that $X_v = Z - Y_v - ce$ is Gaussian with mean and covariance matrix

$$E[X_v] = E[Z] - E[Y_v] - ce \tag{8}$$

$$Cov[X_v,X_v'] = Cov[Z,Z'] + Cov[Y_v,Y_v'] \tag{9}$$

Information from measurements related to X (CPT data)

Let ζ be a Gaussian vector which together with X, Y, X_v, Y_v is jointly Gaussian. It is imagined that ζ is a vector of measurements that contains information about X given through the conditional means and covariances $E[X|\zeta]$ and $Cov[X,X'|\zeta]$. Since the two measuring methods are independent, the information contained in ζ and obtained solely by the CPT method carries no information about the outcome of the measuring error vector Y_v related to the vane test. Therefore all covariances between elements of Y_v and elements of ζ are zero. From the joint Gaussianity and $Cov[Z,Y_v'] = 0$ (modeling assumption 1b) it then follows by use of the linear regression theory apparatus that $Cov[Z,Y_v'|\zeta] = 0$. Moreover it follows that $Cov[Y_v,Y_v'|\zeta] =$

$\mathrm{Cov}[\mathbf{Y}_v,\mathbf{Y}_v']$. Using the information contained in ζ, the mean vector $E[\mathbf{Z}]$ in (8) and the covariance matrix $\mathrm{Cov}[\mathbf{Z},\mathbf{Z}']$ in (9) should therefore simply be replaced by the corresponding conditional quantities $E[\mathbf{Z}|\zeta]$ and $\mathrm{Cov}[\mathbf{Z},\mathbf{Z}'|\zeta]$ respectively. In order to determine these conditional means and covariances we first determine $E[\mathbf{Z}|\mathbf{X}]$ and $\mathrm{Cov}[\mathbf{Z},\mathbf{Z}'|\mathbf{X}]$ by the theory of linear regression of $\mathbf{Z}$ on $\mathbf{X}$. Noting that the homogeneity of $\mathbf{X}$ implies that $E[\mathbf{X}] = E[\mathbf{Z}] = \mu\mathbf{e}$ (setting $E[\mathbf{Y}] = \mathbf{0}$) where μ is the common mean, we have from the previous remark that $E[\mathbf{Z}|\mathbf{X}] = \lambda^2 \mathbf{X} + \mu(1-\lambda^2)\,\mathbf{e}$, $\lambda = \delta/\sigma$, while the residual covariance matrix is $\mathrm{Cov}[\mathbf{Z},\mathbf{Z}'|\mathbf{X}] = \mathrm{Cov}[\mathbf{Z},\mathbf{Z}'] - \mathrm{Cov}[\mathbf{Z},\mathbf{X}']\,\mathrm{Cov}[\mathbf{X},\mathbf{X}']^{-1}\,\mathrm{Cov}[\mathbf{X},\mathbf{Z}'] = \mathrm{Cov}[\mathbf{Z},\mathbf{Z}'] - \lambda^2\,\mathrm{Cov}[\mathbf{Z},\mathbf{Z}'] = \delta^2(1-\lambda^2)\,\mathbf{P}_{\mathbf{X}}$. The total representation theorem next gives the results

$$E[\mathbf{Z}|\zeta] = \lambda^2 E[\mathbf{X}|\zeta] + \mu(1-\lambda^2)\,\mathbf{e} \tag{10}$$

$$\mathrm{Cov}[\mathbf{Z},\mathbf{Z}'|\zeta] = \lambda^2(1-\lambda^2)\,\mathrm{Cov}[\mathbf{X},\mathbf{X}'] + \lambda^4\,\mathrm{Cov}[\mathbf{X},\mathbf{X}'|\zeta] \tag{11}$$

Thus (8) and (9) are replaced by

$$E[\mathbf{X}_v] = E[\mathbf{Z}|\zeta] - c\mathbf{e} \tag{12}$$

$$\mathrm{Cov}[\mathbf{X}_v,\mathbf{X}_v'] = \mathrm{Cov}[\mathbf{Z},\mathbf{Z}'|\zeta] + \gamma^2\,\mathbf{I} \tag{13}$$

setting $E[\mathbf{Y}_v] = \mathbf{0}$. (If $E[\mathbf{Y}]$ and $E[\mathbf{Y}_v]$ are not zero, their contributions may be included in the constant c).

Information from measurements of $\mathbf{X}_v$ (vane test data) leading to the likelihood function of the three unknown parameters c, γ, and λ

Let $\mathbf{x}_v$ be the observation of $\mathbf{X}_v$ and write

$$\xi(c,\lambda) = \mathbf{x}_v - E[\mathbf{X}_v] \tag{14}$$

$$\mathbf{R}(\gamma,\lambda) = \mathrm{Cov}[\mathbf{X}_v,\mathbf{X}_v']^{-1} \tag{15}$$

Then the Gaussian density of $\mathbf{X}_v$ computed at $\mathbf{x}_v$ is

$$f_{\mathbf{X}_v}(\mathbf{x}_v) \propto \sqrt{\det \mathbf{R}(\gamma,\lambda)} \cdot \exp\left[-\tfrac{1}{2}\,\xi(c,\lambda)'\,\mathbf{R}(\gamma,\lambda)\,\xi(c,\lambda)\right], \quad c \in \mathbb{R},\ \gamma \in \mathbb{R}_+,\ \lambda \in [0,1] \tag{16}$$

($\propto$ means "proportional to"). The right side of (16) defines the likelihood function $L(c,\gamma,\lambda;\mathbf{x}_v)$ of c, γ, and λ. Let (C,Γ,Λ) be the set of Bayesian random variables corresponding to (c,γ,λ), and adopt the non–informative prior for which $(C,\log\Gamma,\log(\Gamma^2+\sigma^2\Lambda^2))$ has a diffuse prior over $\mathbb{R} \times \{\log\gamma,\log(\gamma^2+\sigma^2\lambda^2)\,|\,\gamma \in \mathbb{R}_+, 0\leq\lambda\leq 1\}$. Then the prior density of (C,Γ,Λ) is proportional to $\lambda/[\gamma(\gamma^2+\sigma^2\lambda^2)]$ and we get the posterior density $f_{C,\Gamma,\Lambda}(c,\gamma,\lambda|\mathbf{x}_v)$ by multiplying this prior with the likelihood function.

Before the likelihood function (16) can be used to infer about the parameters c, γ, and λ there is a further step to be taken because the set of vane test undrained shear strength observations in practice often will be imperfect in the sense that some of the test results are reported not by their values but by the information that the values are larger than the measuring capacity of the applied vane (censored data). This is expressed by saying that each of the random variables representing the vane test measurements is *clipped* (or censored) at a given value x_{0i}, i = 1, ...,n. Thus the sample is given as $x_1 = x_{01}, x_2 = x_{02}, \ldots, x_r = x_{0r}, x_{r+1} < x_{0r+1}, \ldots, x_n < x_{0n}$, where the sample has been ordered such that the first r vane test shear strengths are larger than the respective measuring capacities while the remaining n − r tests are "well–behaved". For this clipped sampling case the likelihood function is obtained by integrating the joint density of $\mathbf{X}_v$ in (16) with respect to x_i from x_{0i} to ∞ for i = 1, ..., r. Thus the likelihood function becomes

$$L(c,\gamma,\lambda;\mathbf{x}_v) \propto \sqrt{\det \mathbf{R}(\gamma,\lambda)} \cdot \int_{x_{01}}^{\infty} \cdots \int_{x_{0r}}^{\infty} \exp\left[-\tfrac{1}{2}\,\xi(c,\lambda;\mathbf{x})'\,\mathbf{R}(\gamma,\lambda)\,\xi(c,\lambda;\mathbf{x})\right] dx_1 \cdot \ldots \cdot dx_r \tag{17}$$

The numerical studies of the posterior density of (C,Γ,Λ) obtained from (17) after multiplication by the prior density must in practice be based on Monte Carlo integration except if r = 1 or 2.

Specific application to the clay till at Anchor Block West of the Great Belt in Denmark

The mean vectors $E[\mathbf{X}] = \mu\,\mathbf{e}$ and $E[\mathbf{X}|\zeta]$ as well as the covariance matrices $\mathrm{Cov}[\mathbf{X},\mathbf{X}']$ and $\mathrm{Cov}[\mathbf{X},\mathbf{X}'|\zeta]$ have been obtained from the random field modeling of the logarithm of the CPT cone tip resistance q_c for the Anchor Block West area of Storebælt. The details of this modeling is reported in Ditlevsen and Gluver (1991). Interpreting the random field modeling as a pragmatic interpolation tool, Ditlevsen (1991a), the choice of model is made such that it becomes practicable to compute values of the likelihood function (17) a large number of times. Four parameters define the log q_c field. These are μ (mean of field), σ (standard deviation of field) (both used explicitly herein), ρ (vertical correlation parameter), and κ (horizontal correlation parameter). A joint Bayesian distribution of these four parameters has been determined on the basis of 60 CPT cone tip resistance profiles obtained in a rectangular mesh of 6 times 10 points in the horizontal plane with a distance of 20 m between the points in both directions of the mesh. The original cone tip logarithmic resistance profiles have been filtered and averaged in a way as described in Ditlevsen (1991b). The resulting "interpreted" logarithmic CPT profiles are over a depth of 15 m each made up of 150 values that are modeled as an outcome of a homogeneous Gaussian Markov sequence of mean μ, standard deviation σ, and correlation coefficient ρ between successive random variables. In the horizontal planes the correlation structure is modeled such that the correlation coefficient between two interpreted log q_c random variables is $\kappa^{(\mathrm{dist})^2}$ where "dist" is the distance between the two points at which the log q_c values are considered, and dist is measured in the unit of the mesh point distance 20 m.

The statistical evaluation of the parameters c,λ, and γ has been made conditional on given values of μ,σ,ρ, and κ. Due to a relatively small statistical uncertainty of these last parameters their values are first put to their maximum posterior density estimates $\mu = 0.946 + \log[\mathrm{MPa}]$, $\sigma = 0.650 + \log[\mathrm{MPa}]$, $\rho = 0.985$, $\kappa = 0.0156$, Ditlevsen and Gluver (1991).

Fig. 1 shows three sets of contour curves in full line for the posterior joint density of (Λ,Γ) given that C = 2.100, 2.255, 2.400 respectively. The intermediate value is close to the point at which the posterior density of (C,Λ,Γ) is maximal, while the two other values are in the lower and upper tail respectively of the distribution of C, see Fig. 2. The contour curves for the density as obtained by the product of the marginal distributions of Λ and Γ given C = 2.255 are shown by dotted lines. It is seen that (Λ,Γ) is only weakly dependent on C and that there is only a weak dependence between Λ and Γ. In each diagram the marginal densities of Λ and Γ given C = 2.255 are shown along the abscissa axis and ordinate axis respectively.

Fig. 2 shows four conditional density functions for C given (Λ,Γ) = (0.70,0.40), (0.90,0.30), (0.88,0.29) and (0.80,0.25) respectively. It is seen that these distributions are almost identical. This indicates that C in practice can be modeled

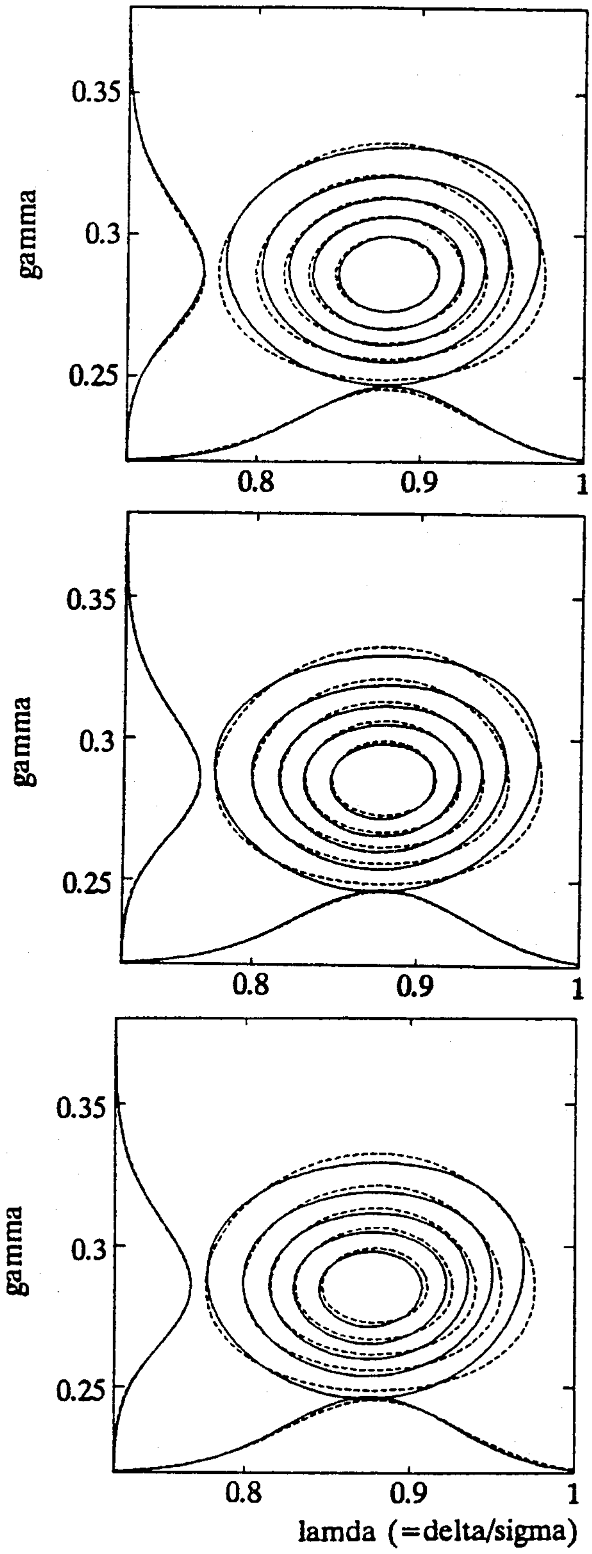

Fig. 1. Contour curves for the joint density of (Λ,Γ) *given that* C = 2.100, 2.255, 2.400 *from top down (full line). The marginal densities of* Λ *and* Γ *are shown along the abscissa axes and the ordinate axes respectively (full lines). The approximating marginal density of* Λ *is shown in the diagrams with dotted line. For* Γ *the approximating density is a truncated Gaussian density. The dotted contour curves correspond to the product of the two marginal densities corresponding to* C = 2.255. *The unit of* exp(Γ) *is* MPa.

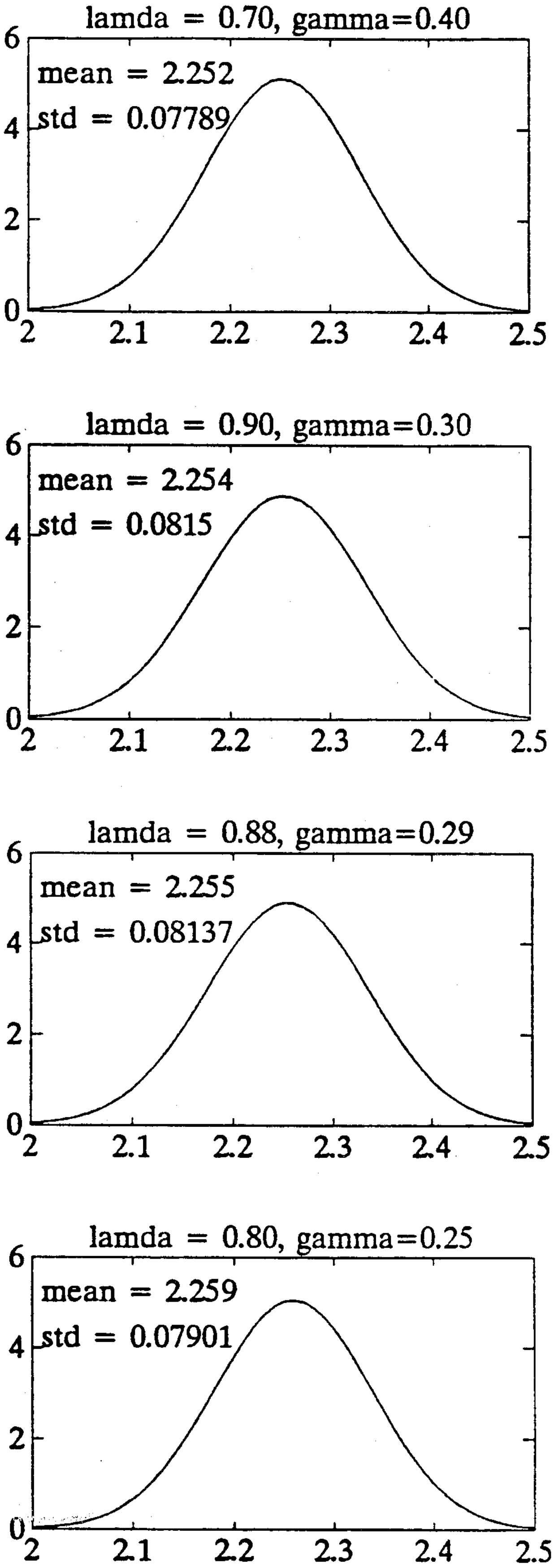

Fig. 2. Conditional density functions for C *given* (Λ,Γ). *These density functions show that* C *is almost independent of* (Λ,Γ). *The unit of* $\exp(\gamma)$ *is* MPa.

to be independent of (Λ,Γ). In conclusion it can be stated that C, Λ, and Γ are practically mutually independent for the given point estimate values of μ,σ,ρ,κ.

The analysis of the posterior density of the Bayesian field parameters M,Σ,P,K (corresponding to μ,σ,ρ,κ respectively) reported in Ditlevsen and Gluver (1991) shows that P can be put equal to a function $\psi_3(\Sigma)$ of Σ. Therefore ρ is eliminated from the numerical studies of the posterior density of (C,Λ,Γ) by substituting $\rho = \psi_3(\sigma)$. Numerical studies with variations of μ,σ,κ in the vicinity of their maximum posterior density estimates show that C depends on μ while Λ depends on σ and κ. Other dependencies are negligible.

With the present data it can be assessed that the parameter C can be modeled to have a Gaussian density with parameters $E[C|\mu] = 1.827 + 0.453\mu$ (unit of $\exp(\mu)$ is MPa), $D[C|\mu] = 0.081$. In particular, $E[C|\mu = 0.946] = 2.255$. With respect to Λ the numerical studies show that the random variable $1/(\Lambda^2+8.05)$ has a probability density that can be approximated by a truncated Gaussian density with mean $a = 10^{-2}[11.34+0.4(\sigma-0.647) + 3(\kappa-0.017)]$ (unit of $\exp(\sigma)$ is MPa), standard deviation $b = 10^{-3}[1.19 + 0.9(\sigma-0.647) - 5(\kappa-0.017)]$, and lower truncation point at $1/(1+8.05) = 0.1105$. Thus Λ^2 can be approximately represented as $[1/(a+bV)]-8.05$ where V is a truncated standard normal random variable with lower truncation point at $(0.1105-a)/b$ ($= -2.439$ for $\sigma = 0.647$, $\kappa = 0.017$) and V is independent of all other random variables in the problem. Fig. 1 (middle) shows the density function for Λ given $C = 2.255$, $\Sigma = 0.647$, $K = 0.017$ together with this approximation.

The results of this numerical study of the likelihood function form the basis for an elaborate reliability analysis of the western anchor block of the Great Belt suspension bridge taking due account of both the measuring uncertainty and the statistical uncertainty as evaluated by the method presented herein, Ditlevsen and Gluver (1991), AS Storebæltsforbindelsen (1991).

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