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Reliability Analysis of Geometrically Nonlinear Structure by Rigid-Plastic Model

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Abstract The self-contradiction in the title of this paper is only apparent because it concerns the application of a particular type of response surface method where the form of the response surface is generated by use of the simple rigid-plastic mechanical theory. The basis is the previously published "model correction factor method". Herein the method is demonstrated to be applicable for efficient and fast reliability analysis of some strongly geometrically nonlinearly behaving frame structures with elastic-plastic constitutive relations.

1. Introduction

Herein the model correction factor method is applied to elastic-plastic frame structures of such slenderness and with such load configurations that geometrically nonlinear effects become important.

The model correction factor method is introduced in Arnbjerg-Nielsen (1991) and is further developed in Ditlevsen and Arnbjerg-Nielsen (1993). It can be classified as a special type of response surface method to be applied to simplify the time consuming reliability analysis when an elaborate mechanical model describes the structural failure behavior. The mathematical class of response surfaces is defined not by some more or less arbitrary class of functions but by a mechanically interpretable model of suitable simplicity. By engineering insight into the actual problem, the simple mechanical model is formulated such that it reflects the most important issues of the elaborate model. However, emphasis is not put on the sophisticated details such as higher order effects that may be built into the elaborate model.

There are several advantages by this type of analysis. One advantage is that the simple model subject to first order reliability analysis directly points at the important domain in the formulation space. Another advantage is that the elaborate fitting procedure in the ordinary response surface method is considerably simplified due to the mechanical information already put into the simple model by its formulation. Moreover the simple model is easily seen through from an engineering point of view making the acceptance of the output from the elaborate model less prone to radical errors or misinterpretations. The method has been succesfully applied in lower bound plastic reliability analysis of damaged concrete decks, Karlsson et al. (1993). In order that this paper can be self-contained the model correction factor method will be summarized in the next section without the argumentations for the validity of the method.

2. Outline of the Model Correction Factor Method

Let (**x**,**y**,**z**) be the total vector of basic variables (input variables) that are contained in the elaborate model. The subvectors **x** and **y** are the vectors of load variables and strength variables respectively. These variables are with sufficient generality defined such that they all have physical units that are proportional to the unit of force. The subvector **z** is the vector of all the remaining basic variables (of type as geometrical and dimensionless basic variables). Two limit state equations $g_r(\mathbf{x,y,z})=0$ and $g_i(\mathbf{x,y,z})=0$ are given representing the elaborate (r for "*realistic*") and the simple (i for "*idealized*") model, respectively. It is assumed that for each fixed $(\mathbf{x,y,z})$ the equations

$$g_r(\kappa_r\mathbf{x,y,z})=0 \quad , \quad g_i(\kappa_i\mathbf{x,y,z})=0 \tag{1}$$

can be solved uniquely with respect to κ_r and κ_i, respectively. The solutions are $\kappa_r(\mathbf{x,y,z})$ and $\kappa_i(\mathbf{x,y,z})$. By using the physical property of dimension homogeneity it is shown in Ditlevsen and Arnbjerg-Nielsen (1993) that the two equations

$$g_r(\mathbf{x,y,z})=0 \quad , \quad g_i(\mathbf{x},\frac{\kappa_r(\mathbf{x,y,z})}{\kappa_i(\mathbf{x,y,z})}\mathbf{y,z})=0 \tag{2}$$

are equivalent in the sense that the two set of points they define are identical. The idea of the model correction factor method is to use a suitably simple approximation to the last equation in (2) in the reliability analysis in place of the first equation in (2). The point is to approximate the function $\nu(\mathbf{x,y,z})=\kappa_r(\mathbf{x,y,z})/\kappa_i(\mathbf{x,y,z})$ by a constant or at most by an inhomogeneous linear function of $(\mathbf{x,y,z})$. The approximation is made such that it is particularly good within the region of the space that contributes the most to the failure probability. Let $(\mathbf{x}^*,\mathbf{y}^*,\mathbf{z}^*)$ be a point of this region and let $\nu^*=\nu(\mathbf{x}^*,\mathbf{y}^*,\mathbf{z}^*)$. The equation

$$g_i(\mathbf{x},\nu^*\mathbf{y,z})=0 \tag{3}$$

then defines an approximating limit state in the important region. The problem is now reduced to the problem of how to choose the point of approximation $(\mathbf{x}^*,\mathbf{y}^*,\mathbf{z}^*)$. The answer to this problem is given in the reliability theory. With a judgementally chosen value ν_0 of ν^* a first or second order reliability analysis (FORM or SORM, see eg. Madsen, Krenk and Lind (1986) or Ditlevsen and Madsen (1991)) is made with (3) as limit state. This analysis determines the most central point (the design point) $(\mathbf{x}_1,\mathbf{y}_1,\mathbf{z}_1)$ and an approximate failure probability p_1. Using that $\kappa_i(\mathbf{x}_1,\mathbf{y}_1,\mathbf{z}_1)=1/\nu_0$ an improved value $\nu_1=\nu_0\kappa_{r1}$ of ν^* is calculated where $\kappa_{r1}=\kappa(\mathbf{x}_1,\mathbf{y}_1,\mathbf{z}_1)$. Then a new FORM or SORM analysis is made with (3) as limit state. This gives the most central point $(\mathbf{x}_2,\mathbf{y}_2,\mathbf{z}_2)$ and an approximate failure probability p_2. Proceeding iteratively in this way we get a sequence $(\kappa_{r1},p_1),(\kappa_{r2},p_2),\ldots$ that may or may not be convergent. If the sequence is convergent in the first component it is also convergent in the second component and we have $\kappa_{r1},\kappa_{r2},\ldots\rightarrow 1$, $p_1,p_2,\ldots\rightarrow p$, where p will be denoted as the *zero order approximation* to the probability of the failure event of the elaborate model.

If the sequence is not convergent we still can define the zero order approximation by simple interpolation to the value $\kappa_r=1$ among points (κ_r,β) ($\beta=-\Phi^{-1}(p)$, Φ = standardized normal distribution function) corresponding to the sequence or simply obtained for a series of different values of ν_0.

A check of the goodness of the zero order approximation is made by replacing the function $\nu(\mathbf{x,y,z})$ by its first order Taylor expansion

$$\nu(\mathbf{x},\mathbf{y},\mathbf{z}) \approx \tilde{\nu}(\mathbf{x},\mathbf{y},\mathbf{z}) = \nu^* + \mathbf{a}^T(\mathbf{x}-\mathbf{x}^*) + \mathbf{b}^T(\mathbf{y}-\mathbf{y}^*) + \mathbf{c}^T(\mathbf{z}-\mathbf{z}^*) \tag{4}$$

at the most central point $(\mathbf{x}^*,\mathbf{y}^*,\mathbf{z}^*)$ corresponding to the limit state (3) with ν^* being the value corresponding to $\kappa_r=1$. The numerical determination of the coefficients **a**, **b** and **c** requires that the values of $\nu(\mathbf{x},\mathbf{y},\mathbf{z})$ are known at least at as many points in the vicinity of $(\mathbf{x}^*,\mathbf{y}^*,\mathbf{z}^*)$ as the number of variables in $(\mathbf{x},\mathbf{y},\mathbf{z})$. These values of ν are obtained by solving the equations (1) with respect to κ_r and κ_i respectively at each chosen point $(\mathbf{x},\mathbf{y},\mathbf{z})$.

With (4) substituted for κ_r/κ_i into the last equation in (2) we get a limit state for which both the probability of failure and the value of κ_r in general will be different from p and the value $\kappa_r=1$ as obtained by the zero order approximation. However, by a unique scaling factor k_r on the load vector **x** we can achieve that the limit state $g_i(k_r\mathbf{x},\nu(k_r\mathbf{x},\mathbf{y},\mathbf{z})\mathbf{y},\mathbf{z})=0$ corresponds to the failure probability p. With the Taylor expansion (4) substituted into this equation we get the limit state equation $g_i(k_r\mathbf{x},\tilde{\nu}(k_r\mathbf{x},\mathbf{y},\mathbf{z})\mathbf{y},\mathbf{z})=0$ for which we can determine k_r by iterative application of FORM or SORM analysis such that the corresponding failure probability becomes p. The size of the deviation of k_r from 1 can then be used to judge the accuracy of the zero order approximation. Also the change of the most central point contributes to this judgement.

In case k_r deviates too much from 1 an iterative procedure can be applied just as for the zero order approximation to obtain $k_r=1$ in the limit and a corresponding improved value of p. This value will be called the *first order approximation.*

Remark. In the particular case where the joint distribution of the random load vector **X** is Gaussian it is possible to give a simple interpretation of the deviation of k_r from 1. With **U** being a standard Gaussian vector we have $\mathbf{X}=\mathbf{M}[(\mathbf{M}^{-1}\text{Cov}[\mathbf{X},\mathbf{X}^T],\mathbf{M}^{-1})^{1/2}\mathbf{U}+\mathbf{e}]$ where **M** is the diagonal matrix with the elements of $\boldsymbol{\mu}=E[\mathbf{X}]$ as diagonal elements, $\mathbf{e}^T=[1...1]$, and the symbol $\mathbf{A}^{1/2}$ stands for any matrix that satisfies $\mathbf{A}^{1/2}(\mathbf{A}^{1/2})^T=\mathbf{A}$. If $\mathbf{M}^{-1}\text{Cov}[\mathbf{X},\mathbf{X}^T]\mathbf{M}^{-1}=\{\rho[X_i,X_j]V_{Xi}V_{Xj}\}$ is independent of **M** (in one dimension this simplifies to a fixed coefficient of variation) it follows from the above representation of **X** that the reliability of the limit state $g_r(k_r\mathbf{x},\mathbf{y},\mathbf{z})=0$ is the same as the reliability of the limit state $g_r(\mathbf{x},\mathbf{y},\mathbf{z})=0$ given that the mean of **X** is simply changed from $\boldsymbol{\mu}$ or $k_r\boldsymbol{\mu}$. In this consideration it is assumed that the random vectors **Y** and **Z** are not dependent on $\boldsymbol{\mu}$. This interpretation is used herein for the graphical presentation of the first order approximation results (Figure 3).

3. Carrying Capacity Calculation

The elaborate model is a geometrically nonlinear elasto-plastic FE-model (appendix A). The material is assumed to be linear elastic with strain hardening after the onset of yielding (appendix B). The carrying capacity for the frame structure in the FE-model is defined to be the first load parameter maximum on the load-displacement curve. A load controlled incrementation is capable of tracing the load-displacement curve to a point close to the first load maximum. This point is arbitrarily determined as the point at which the iteration convergence fails. This arbitrariness of the point causes the corresponding numerical derivatives needed for calculating the Taylor expansion of the correction factor $\nu(\mathbf{x},\mathbf{y},\mathbf{z})$ in the reliability analysis calculations to be too unstable to be useful. The problem is eliminated using displacement controlled incrementation, appendix C, allowing load maxima to be calculated more accurately giving sufficiently stable numerical derivatives.

The simple model is a rigid-plastic yield hinge model in which the yield criterion includes moments only. Potential yield hinges are placed at supports, in frame corners and at points with concentrated applied loads. The orthogonal corner frame example considered in the following shows that it can be necessary to have yield hinges at some few other points of the frame. The cross sectional ideal plastic moment capacity is given as 1/4 WD^2 where W and D are the width and depth, respectively, of the assumed massive rectangular beam cross sections.

4. Reliability Analysis

The reliability analyses are carried out on a HP 9000/730 using the general purpose probabilistic analysis program package Proban version 3.0 (Det Norske Veritas Research). In the model correction factor based analysis the reliability calculations are related to the collapse modes of the simple model. Herein only the design point for the collapse mode with lowest reliability as determined by FORM is considered. Thus the obtained reliability estimated in the analysis is the reliability corresponding to the probabilistically most critical collapse mode. Often this represents a good approximation to the system reliability. If the system reliability were to be considered directly it would be necessary to assign a correction factor to the design point for each collapse mode in the simple model or at least to some of the most important of these.

In the examples herein the limit state equations for the elaborate and the simple model get the appearance $-\kappa_r+\lambda_r(\mathbf{x},\mathbf{y},\mathbf{z})=0$ and $-\kappa_i+\lambda_i(\mathbf{x},\mathbf{y},\mathbf{z})=0$ respectively. The functions $\lambda_r(\cdot)$ and $\lambda_i(\cdot)$ are the carrying capacity functions in each of the two models. This particular form of the limit state equations gives the simplification relative to the general problem that solving with respect to κ_r and κ_i only requires one calculation of $\lambda_r(\mathbf{x},\mathbf{y},\mathbf{z})$ and $\lambda_i(\mathbf{x},\mathbf{y},\mathbf{z})$.

5. Examples: Portal Frame and Orthogonal Corner Frame

Reliability analyses are made for a portal frame, Figure 1 (left), L=10.0 m and H=7.0 m. In order to exaggerate geometrically nonlinear effects the frame is made relatively slender. The dimensions of all cross sections (appendix B) are fixed to depth D=0.2959 m and to width W=0.02639 m. The Youngs modulus is fixed to $E=2.10\times10^5$ MPa. The hardening parameter n in (B.1) is fixed to 10. In the elaborate model each of the substructures AB, BC, CD, and DE are subdivided into 6, 5, 5, and 6 beam elements respectively (appendix B). The finite element model has not been verified by a model with more elements. All probabilistic data of the problem are listed in Table 1.

Table 1. Data for reliability analysis of the portal frame (C.o.V. = Coefficient of variation)

Name	Distribution	Description	Mean	C.o.V.
F_{HB}	Normal	Applied horizontal force at point B	μ_F	0.2
F_{VB}	Normal	Applied vertical force at point B	$\eta\mu_F$	0.2
F_{VC}	Normal	Applied vertical force at point C	μ_F	0.2
f_{yPQ}	Lognormal	Yield stress in PQ (=AB, BD or DE)	1500 MPa	0.1

f_{yPQ} are equi-correlated with correlation coefficient 0.3. No other correlation.

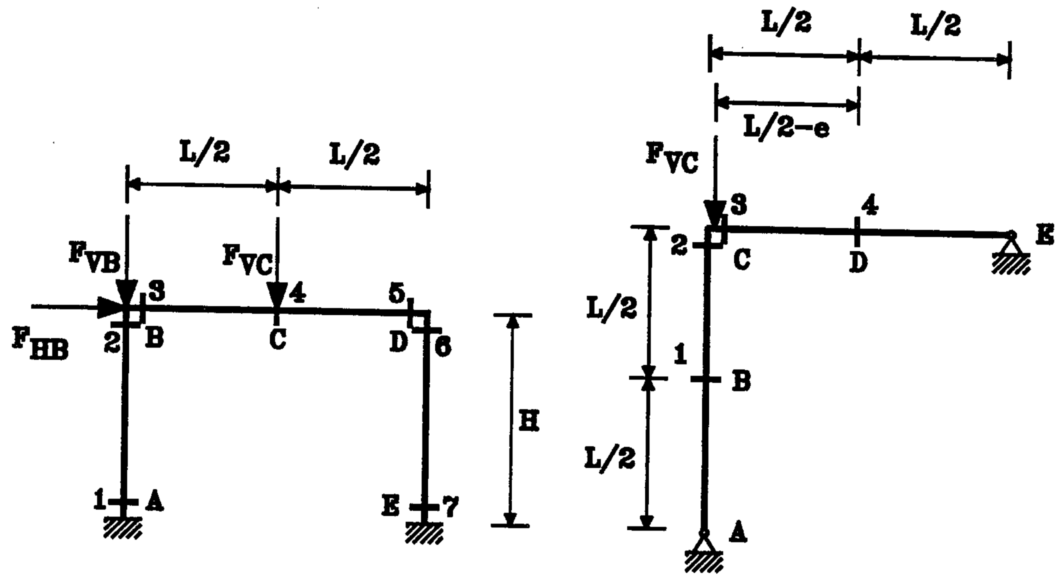

Figure 1. Portal frame (left) and orthogonal corner frame (right). Numbers refer to yield hinge positions

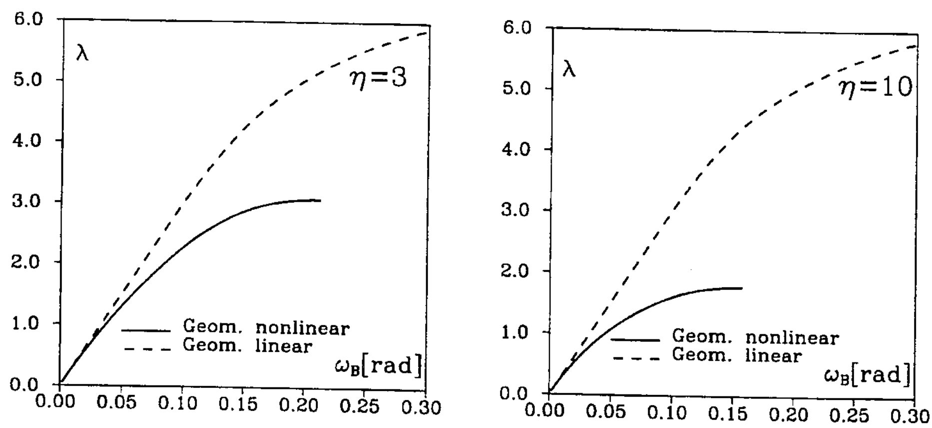

Figure 2. Load parameter λ versus clockwise rotation ω_B of B of the portal frame (controlled displacement, appendix C) for η=3 (left) and η=10 (right)

The reliability analysis is carried out for η=3, Table 1. Different degrees of geometrical nonlinearity can be illustrated by deterministic analyses for different load configuration cases, i.e. different values of η. Load-displacement curves corresponding to geometrically linear and geometrically nonlinear behavior are plotted for η=3 and η=10 in Figure 2. The ordinate λ is the load factor applied to the load configuration such that it balances an imposed rotation ω_B of the frame corner B. For η=0 the effect from the geometrical nonlinearity is very moderate. Otherwise it is seen that the effect from the geometrical nonlinearity is important but that the effect varies relatively slowly with η and thus with F_{VB}. Although F_{VB} has no influence in the simple yield hinge model, the model correction factor method turns out to be able to capture the overall changed mechanical behavior of the structure caused by the rigidity decreasing effect of F_{VB}, Figure 3 (left).

The orthogonal corner frame is shown in Figure 1 (right), L=10.0 m. The cross sectional geometrical properties and the Youngs modulus are as for the portal frame. Ideal plasticity is assumed, i.e. the hardening parameter n in (B.1) is put to infinity (a simplified version of (B.1) is used in this case). In the elaborate model each of the substructures AC and CE are subdivided into 5 beam elements. The finite element model has not been verified by a model with more elements. All probabilistic data of the problem are given in Table 2.

Table 2. Data for reliability analysis of the orthogonal corner frame (C.o.V. = Coefficient of variation)

Name	Distribution	Description	Mean	C.o.V.
F_{VC}	Normal	Applied vertical force at point C	μ_F	0.2
e	Normal	Horiz. eccentricity of F_{VC} (pos. C→D)	0.1 m	0.2
f_{yRS}	Lognormal	Yield stress in RS (=AC or CE)	400 MPa	0.1

f_{yAC} and f_{yCE} are correlated with correlation coefficient 0.3. No other correlation.

The probabilistically most critical mechanism as determined by FORM (Section 4) is found among 10 and 5 mechanisms for the portal and the orthogonal corner frame respectively. It is noted that the standard formulation of a rigid-plastic yield hinge model for the orthogonal corner frame as considered herein only contains two yield hinges both placed in the horizontal beam, the one at the corner and the other at the load. However, this standard model is unsufficient here because the corresponding beam mechanism is not in any way resembling the elasto-plastic displacement field implied by the elaborate model. In fact one should first establish a reasonable elastic-plastic hinge model that crudely behaves like the elaborate model with respect to geometrical nonlinearity. Thus there obviously should be a reasonable number of separated rotation hinges in the model. Next step is the drastic one to let these hinges be rigid-plastic. The justification of this step relies on the limit state defined by the mechanism equations to be of a reasonable mathematical similarity with the limit state of the elaborate model. At least this is needed in the probabilistically most important region of the formulation space.

6. Results

Results from the correction factor based analyses are compared in Figure 3 to results from direct FORM analyses using the elaborate model. The results from the FORM analyses have been verified by directional simulation (not shown).

It is seen that the first order approximation result based on the model correction factor method gives an excellent approximation to the reliability index. The CPU time used for the FORM analysis was of the order of 1 hour for each point in Figure 3, increasing with increasing reliability index. With a reasonable starting value for the zero order model correction factor, the zero order and the first step of the first order correction factor analysis took only some few minutes in CPU time per point. The iteration to the final first order approximation result was of the order 10 minutes in CPU time.

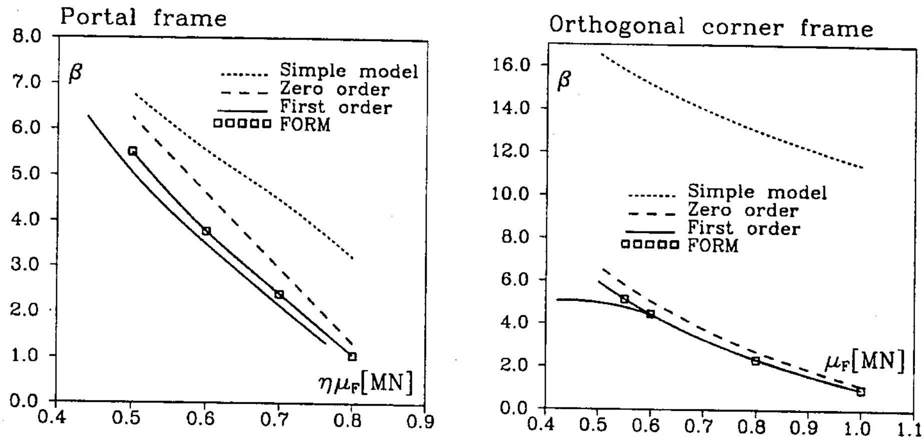

Figure 3. Reliability index $\beta=-\Phi^{-1}(p)$ versus mean load level. From below the full curves correspond to the first step and the final step of the first order approximation calculation respectively.

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References

Arnbjerg-Nielsen, T (1991): *Rigid-ideal Plastic Model as a Reliability Analysis Tool for Ductile Structures*. Ph.D.-thesis, Department of Structural Engineering, Technical University of Denmark, Series R, No 270.

Byskov, E. (1982-83): *Plastic Symmetry of Roorda's Frame*. Journal of Engineering Mechanics, 10(3), 311-328.

Clarke, M.J. and Hancock (1990), G.J.: *A Study of Incremental-Iterative Strategies for Non-linear Analyses*. International Journal for Numerical Methods in Engineering, Vol 29, 1365-1391.

Ditlevsen, O. and Madsen, H.O. (1990): *Bærende konstruktioners sikkerhed* (in Danish, under translation to English). SBI-rapport 122.

Ditlevsen, O. and Arnbjerg-Nielsen,T. (1993): *Model Correction Factor in Structural Reliability*. Journal of Engineering Mechanics, ASCE.

Forde, B.W.R. and Stiemer, S.F. (1987): *Improved Arc Length Orthogonality Methods for Nonlinear Finite Element Analysis*. Computers and Structures 27(5).

Gierlinsky, J.T. and Graves Smith, T.R. (1985): *A Variable Load Iteration Procedure for Thin-Walled Structures*. Computers and Structures Vol. 21, No. 5, pp. 1085-1094.

Karlsson, M., Johannesen, J.M. and Ditlevsen, O. (1993): *Reliability Analysis of an Existing Bridge*. IABSE Colloquium: Remaining Structural capacity, Copenhagen. IABSE Report Vol. 67, pp. 19-28.

Madsen, H.O., Krenk, S. and Lind, N.C. (1986): *Methods of Structural Safety*. Prentice-Hall.

Powell, G. and Simons, J. (1981): *Improved Iteration Strategy for Nonlinear Structures*. International Journal for Numerical Methods in Engineering, Vol 17, 1455-1467.

Appendix A: Finite Element Formulation

A Lagrangian strain measure is employed. The generalized strain components are the axial strain ε and the curvature κ given by

$$\varepsilon = \frac{du}{dx} + \frac{1}{2}\left(\frac{dw}{dx}\right)^2 \ ; \qquad \kappa = -\frac{d^2 w}{dx^2} \tag{A.1}$$

where u and w are the axial and transverse displacements, respectively. The fiber strains in a cross section are assumed to vary linearly with depth. The finite element equations are based on an incremental principle of virtual work assuming proportional loading, Byskov (1982-83). The implemented incremental finite element equations (before assembling of elements) are

$$\begin{aligned}&[\int \mathbf{B}^T\mathbf{D}\mathbf{B}\,dV + \int \mathbf{B}^T\mathbf{d}_1\mathbf{u}_0^T\mathbf{C}\,dV + \int \mathbf{C}\mathbf{u}_0\mathbf{d}_1^T\mathbf{B}\,dV + \int d_{11}\mathbf{C}\mathbf{u}_0\mathbf{u}_0^T\mathbf{C}\,dV + \int \mathbf{C}\sigma^f_0\,dV\,]\Delta\mathbf{u} \\ &= \lambda \int \mathbf{N}^T\mathbf{t}\,dK - [\int(\mathbf{b}_1 + z\mathbf{b}_2)\sigma^f_0\,dV + \int \mathbf{C}\sigma^f_0\,dV\mathbf{u}_0]\end{aligned} \tag{A.2}$$

where

0 as subscript refers to the previous state
Δ = operator referring to increment from the previous state to the current state
$\mathbf{D}$ = matrix that defines the tangential constitutive relations between generalized stresses and strains
$\mathbf{d}_1^T$ = first row of $\mathbf{D}$
d_{11} = element in first row and first column of $\mathbf{D}$
σ^f = fiber stress
$\mathbf{N}$ = displacement distribution matrix
$\mathbf{t}$ = load configuration vector
λ = load parameter
$\mathbf{u}$ = displacement vector
$\mathbf{B}$ = linear strain distribution matrix; $\varepsilon_{linear}=\mathbf{u}^T\mathbf{B}\mathbf{u}$; $\varepsilon=\varepsilon_{linear}+\varepsilon_{nonlinear}$; $\varepsilon^T=[\varepsilon\ \kappa]$
$\mathbf{b}_1^T$ = first row of $\mathbf{B}$
$\mathbf{b}_2^T$ = second row of $\mathbf{B}$
$\mathbf{C}$ = nonlinear strain distribution matrix, i.e. $\varepsilon_{nonlinear}=1/2\ \mathbf{u}^T\mathbf{C}\mathbf{u}$
z = coordinate transverse to the element axis
dV = infinitesimal volume element
dK = infinitesimal boundary element

Appendix B: Beam Finite Element and Strain Hardening Material

The finite element is defined as a three-noded element as shown in Figure B.1. In order to get a linear contribution to the axial strain from the axial displacement a node allowing axial displacement is placed at the centre point of the element. The element stiffness matrix and the nodal forces corresponding to given stress distributions are calculated using Gaussian quadrature with integration points along the element axis. Membrane locking phenomena are avoided by only

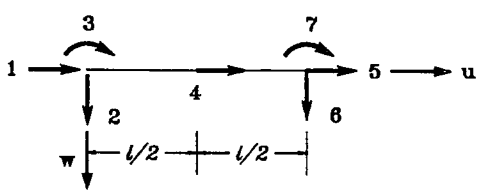

Figure B.1. Beam finite element

using two Gauss points. In the present examples the cross sections are massive rectangular. Integration over the beam cross section is made by Gaussian quadrature. Five integration points along the depth give sufficient accuracy.

The structure is linear elastic up to yielding with Youngs modulus E and initial yielding stress f_{yi}. Strain hardening is assumed according to (B.1). The parameter n=1 corresponds to linear elastic behavior while as n tends to infinity the behavior becomes linear elastic ideal-plastic.

$$\frac{d\varepsilon}{d\sigma} = \frac{1}{E} \qquad \text{for } (|\sigma| \leq f_{yi}) \vee [(|\sigma|) \geq f_{yi}) \wedge (\sigma \Delta \sigma \leq 0)]$$

$$\frac{d\varepsilon}{d\sigma} = \frac{1}{E}\left(\frac{|\sigma|}{f_{yi}}\right)^{n-1} \qquad \text{for } (|\sigma| > f_{yi}) \wedge (\sigma \Delta \sigma > 0) \tag{B.1}$$

Appendix C: Load Incrementation and Equilibrium Iteration

The loading increment is controlled by a pre-chosen single displacement component in the structure, eg. Powell and Simons (1981). The method requires that the controlling displacement varies monotonously with the loading parameter.

At each loading level the controlling displacement is kept constant while iterations are performed in order to obtain a state of equilibrium. The equilibrium iteration follows a modified Newton-Raphson iteration scheme, except that in order to meet the constant displacement requirement the loading from the unbalanced nodal forces **g** in the system are supplemented with an extra loading proportional to the load configuration **t**. This determines the load parameter increment $\Delta\lambda$ according to the equation

$$\mathbf{h}^T\mathbf{K}^{-1}(\Delta\lambda\mathbf{t}+\mathbf{g})=0 \tag{C.1}$$

The matrix **K** is the tangential system stiffness matrix while the vector **h** relates the controlling displacement $v_{control}=\mathbf{h}^T\mathbf{v}$ to the system displacement vector **v**.

In the present study it is easy to choose displacement components that for relevant realizations of the basic variables fulfill the monotony requirement (eg. rotation of B for the portal frame and rotation of C for the orthogonal corner frame). If such displacement components are difficult to identify there are other possibilities. These are methods like the well known constant arc length method, eg. Forde and Stiemer (1987), or the constant weighted response method, Gierlinsky and Graves Smith (1985), of which the former method can be considered as a special case. For a comprehensive comparative study on a large number of methods reference is made to Clarke and Hancock (1990).

The carrying capacity is defined as the first obtained local maximum of the load parameter during increasing (controlled) displacement from zero. Stop is set after the passage of the maximum as soon as the load parameter becomes below a chosen fraction k ($0<k<1$) of the obtained maximum value. In order to prevent a break down of the FORM analyses and the simulations using the FE-model directly, a maximum number of displacement steps was set to be used in each carrying capacity calculation. This limitation has caused that the carrying capacity is underestimated in some few cases.