LIKELIHOOD ESTIMATION OF PARAMETERS USING SIMULTANEOUSLY MONITORED PROCESSES

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Abstract

The topic is maximum likelihood inference from several simultaneously monitored response processes of a structure to obtain knowledge about the parameters of other not monitored but important response processes when the structure is subject to some Gaussian load field in space and time. The considered example is a ship sailing with a given speed through a Gaussian wave field.

Introduction

The ship hull of a modern ship is usually equipped with a few sensors that monitor the stresses and the accelerations at selected locations. The received signals guide the captain during loading and unloading and during voyage to decide on speed reduction or on changing the course. Available knowledge about the ship structure is given in the form of a mathematical structural model and a stochastic description of the load and response parameters. With access to fast information technology it is possible to utilize the current inflow of information embedded in the set of simultaneously monitored data series to make immediate Bayesian updating of any reasonably chosen prior probability and spectral distributions.

The basic principles of the topic are treated in (Friis-Hansen & Ditlevsen 2003) dealing with the method of maximum likelihood estimation of spectral parameters of a stationary Gaussian process. Herein a specific structure example is considered for illustrating Bayesian inference from several simultaneously monitored response processes to obtain knowledge about other not monitored but important response processes when the structure is subject to some Gaussian load field in space and time. The considered example is a ship sailing with a given speed through a Gaussian wave field of unknown spectral parameters. We therefore start out by summarizing the fundamentals of the so-called strip load model which is a strongly simplified model of the hydrodynamic forces on the ship hull.

Summary of Strip Load Model

Consider a ship with symmetric cross-sections described in an (x, y, z)-coordinate system which is fixed relative to the ship with the x-axis along the center line of the ship pointing in the front direction (bow), the y-axis directed to the right (starboard), and the z-axis directed vertical up. In calm water the (x, y)-plane is horizontal and coincident with the water surface. Let the ship move with velocity V along a straight line under the angle β

with a straight ξ-axis pointing in the direction of the propagation of a plane purely harmonic unit amplitude Airy wave $h(\xi, t, \omega) = \exp[i(k\xi - \omega t)]$. Thus $\xi = (x + Vt)\cos\beta - y\sin\beta$. The wave frequency ω and the wave number k are assumed to be related to each other through the deep water dispersion relation $\omega^2 = kg$, in which g is the gravity acceleration constant.

Besides the forward movement the ship moves up an down due to the wave excitation. The vertical component of the movement generates a varying pressure on the ship hull partly caused by the passive buoyancy and the wave generated pressure variations and partly by the active ship movement generated accelerations of the water body around the ship. The vertical component of the first force part per length unit of the ship $F_s(x, t)$ (s for static) is called the Froude-Krylov force. Elementary hydrostatic and linear wave theory considerations lead to the first order approximation

$$F_s(x, t, \omega) = \rho g[A_0(x) - B_0(x)\tilde{z}(x, t, \omega)], \quad \tilde{z}(x, t, \omega) = u(x, t) - \kappa(x, \omega)h(\xi, t, \omega) \qquad (1)$$

where $A_0(x)$ is the area of the part of the cross-section at position x for which $z < 0$, $B_0(x)$ is the width of the hull at x for $z = 0$, ρ is the mass density of the ocean water, $u(x, t)$ is the elevation of the coordinate plane $z = 0$ above the calm water level, and $\kappa(x, \omega) = 1 - k\int_{-T}^{0} e^{kz}B(x, z)\,dz / B_0(x)$ is the so-called Smith correction factor. $B(x, z)$ is the width of the hull at position (x, z) and $-T$ is the z-coordinate of the bottom of the hull, that is, T is the draft in still water.

The second force part is much more difficult to model. The so-called linear strip theory leads to the uttermost simplified but surprisingly realistic hydrodynamic force model

$$F_d(x, t, \omega) = -\frac{D}{Dt}\left[M(x, \omega_e)\frac{D}{Dt}\tilde{z}(x, t, \omega)\right] - N(x, \omega_e)\frac{D}{Dt}\tilde{z}(x, t, \omega) \qquad (2)$$

$$\omega_e = |\omega - \omega^2/(2\omega_0)|, \quad \omega_0 = g/(2V\cos\beta) \qquad (3)$$

in which the derivative D/Dt is the so-called total time derivative by which $D\tilde{z}/Dt = -V\partial\tilde{z}/\partial x + \partial\tilde{z}/\partial t$. The frequency ω_e is the so-called encounter frequency that comes from the Doppler frequency shift caused by the speed V of the ship under the angle β with the wave direction. The function $M(x, \omega_e)$ is the added mass function, and $N(x, \omega_e)$ is a damping function. These functions are defined on the basis of an idealized hydrodynamic problem in which an "infinitely" long cylinder of cross-section as the hull cross-section at x and placed in the water as the ship. The cylinder is subject to a vertical unit amplitude harmonic movement of frequency ω_e. The vertical force needed to maintain the harmonic movement and the phase shift are measured. From the measurement results it is then possible to calculate values for $M(x, \omega_e)$ and $N(x, \omega_e)$. In the linear theory there is the relation $\int_0^\infty \{\omega[M(x, \omega) - M(x, \infty)]\sin\omega t + N(x, \omega)\cos\omega t\}\,d\omega = 0$ between the added mass in excess of $M(x, \infty)$ and the damping function $N(x, \omega)$. In fact, from the Fourier integral theory it follows that this relation determines the one from the other. Of course, the relation should be taken into account in the data fitting. Hydrodynamic considerations and numerical calculations may also lead to a representative function family within which $M(x, \omega)$ may be approximately represented by fitting of the unknown parameters to the measured data. This paper contributes to the solution of this parameter fitting problem on the basis of measurements of the movements of the ship in operation. For details concerning the presented linear strip load model the reader is referred to (Jensen 2001).

Ship Motion

Under the excitation from the unit amplitude wave $h(\xi, t, \omega)=\exp[i(k\xi-\omega t)]$ the rigid body motion of the ship in the (x, z)-plane is a harmonic motion of frequency as the encounter frequency ω_e and with amplitude $U_0(\omega_e, k; V, \beta) + U_1(\omega_e, k; V, \beta)(x - x_g)$ where x_g is the x-coordinate of the center of gravity of the ship, and where

$$-\omega_e^2 U_0(\omega_e, k; V, \beta) \int_0^L m(x)\,dx = e^{-i\omega_e t} \int_0^L F(x, t, \omega)\,dx \tag{4}$$

$$-\omega_e^2 U_1(\omega_e, k; V, \beta) \int_0^L [m(x)(x-x_g)+J(x)]\,dx = e^{-i\omega_e t} \int_0^L (x-x_g) F(x, t, \omega)\,dx \tag{5}$$

with $F(x, t, \omega)=F_s(x, t, \omega)+F_d(x, t, \omega)$ where the two terms are given by (1) and (2), respectively, with

$$\tilde{z}(x, t, \omega) =$$
$$[U_0(\omega_e, k; V, \beta) + U_1(\omega_e, k; V, \beta)(x-x_g)-K(x, \omega)\,\kappa(x, \omega)e^{-ik\cos\beta\, x}]\,e^{i\omega_e t} \tag{6}$$

in which $K(x, \omega) = B_0(x)^{-1} \int_{-B_0(x)/2}^{B_0(x)/2} e^{ik\sin\beta\, y}\,dy = \sin[k\sin\beta\, B_0(x)/2]/[k\sin\beta\, B_0(x)/2]$. Thus (4) and (5) give two equations with the amplitudes $U_0(\omega_e, k; V, \beta)$ and $U_1(\omega_e, k; V, \beta)$ as unknowns. These are two modal transfer functions in the sense that if the excitation is a superposition of harmonics $\int_0^\infty e^{i\omega_e t}\,dW_e(\omega_e)$ then the rigid body motion of the ship is the real part of the superposition $u_0(t; V, \beta)+u_1(t; V, \beta)(x-x_g)$ where

$$u_j(t; V, \beta) = \int_0^\infty e^{i\omega_e t} U_j(\omega_e, k; V, \beta)\,dW_e(\omega_e), \quad j = 0, 1 \tag{7}$$

Gaussian Wave Excitation

Assume now that $W_e(\omega_e)$ is a process of stationary and independent zero mean Gaussian increments on the ω_e-axis. Then (suppressing V and β)

$$\mathrm{Cov}[u_j(0), u_k(t)] = 2 \int_0^\infty e^{-i\omega_e t}\, U_j(\omega_e, k)U_k^*(\omega_e, k)\, S_e(\omega_e)\,d\omega_e, \quad j, k = 0, 1 \tag{8}$$

where $S_e(\omega_e)$, denoted the encounter spectrum, is the spectral density function of the real as well as the imaginary part of the complex process $W_e(\omega_e)$, that is, $E[dW(\omega_{e1})\,dW(\omega_{e2})] = 2\delta(\omega_{e1} - \omega_{e2})\, S_e(\omega_{e2})\,d\omega_{e1}\,d\omega_{e2}$ and $E[dW(\omega_e)\,dW^*(\omega_e)] = 2S_e(\omega_e)\,d\omega_e$, where $\delta(\cdot)$ is Dirac's delta function.

To obtain the encounter spectrum $S_e(\omega_e)$ in terms of the given wave elevation spectrum $S(\omega)$ we note that ω_e given by (3) is not a monotonic function of ω when $\omega_0 > 0$. In fact, for $0 \leq \omega_e \leq \omega_0/2$ there are three values of ω that give the same encounter frequency. These are $\omega_1 = \omega_0 - \sqrt{\omega_0(\omega_0 - 2\omega_e)}$, $\omega_2 = \omega_0 + \sqrt{\omega_0(\omega_0 - 2\omega_e)}$, and $\omega_3 = \omega_0 + \sqrt{\omega_0(\omega_0 + 2\omega_e)}$. For $\omega_0/2 \leq \omega_e$ only ω_3 gives ω_e. For $\omega_0 < 0$ only ω_2 gives ω_e. Thus

$$S_e(\omega_e) = \sum_{\text{relevant } j} S(\omega_j) \left| \frac{d\omega}{d\omega_e} \right|_{\omega=\omega_j} = \sum_{\text{relevant } j} S(\omega_j) \left| \frac{\omega_0}{\omega_j - \omega_0} \right|, \quad \omega_j \neq \omega_0 \tag{9}$$

Gaussian Trigonometric Polynomial

To be able to operate numerically with any of the considered Gaussian processes they are replaced by trigonometric polynomials of the form

$$Q(t) = \sum_{k=1}^{N} [\Delta W_r(\omega_k) \cos \omega_k t + \Delta W_i(\omega_k) \sin \omega_k t] \tag{10}$$

where $\omega_k = k\Delta\omega$, $\Delta W_r(\omega_k)$ and $\Delta W_i(\omega_k)$ for all $k = 1, \ldots, N$ are mutually independent Gaussian random variables of zero mean and standard deviation $\sqrt{S(\omega_k)\Delta\omega}$. This corresponds to the discretization $0 < \Delta\omega < 2\Delta\omega < \ldots < N\Delta\omega$. The discretization is reasonably chosen such that numerical integration of the spectrum $S(\omega)$ over the interval $[0, N\Delta\omega]$ by the trapezoidal formula, say, and the chosen discretization deviates less than some chosen small value from the exact variance $\int_0^\infty S(\omega) \, d\omega$.

Since the functions $\cos \omega_k t, \sin \omega_k t, k = 1, \ldots, N$, define an orthogonal function system over the interval $[0, T]$, where $T = 2\pi/\Delta\omega$ is the period of the trigonometric polynomial (10), it follows directly that

$$\Delta W_r(\omega_k) = \frac{2}{T} \int_0^T Q(\tau) \cos \omega_k \tau \, d\tau, \quad \Delta W_i(\omega_k) = \frac{2}{T} \int_0^T Q(\tau) \sin \omega_k \tau \, d\tau \tag{11}$$

Maximum Likelihood Estimation of Spectral Parameters

Let $\mathbf{Q}(t)$ be a vector of random trigonometric polynomials $Q_1(t), \ldots, Q_n(t)$ as (10). Assume that a sample $\mathbf{q}(t)$ of $\mathbf{Q}(t)$ has been observed over the time interval $[0, T]$. By substituting $\mathbf{q}(\tau)$ for $Q(\tau)$ in (11) the corresponding realizations of the polynomial coefficients are obtained. The covariance matrix of the coefficients to $\cos \omega_k t$ or to $\sin \omega_k t$ is $\mathbf{S}(\omega_k)\Delta\omega = \{S_{rs}(\omega_k)\}_{nn}\Delta\omega$ where $S_{rs}(\omega)$ is the cross-spectrum (or auto-spectrum if $r = s$) between $Q_r(t)$ and $Q_s(t)$. Let a family of spectral matrices $\mathbf{S}(\omega; \boldsymbol{\mu})$ be given a priori, where $\boldsymbol{\mu}$ is an unknown vector of parameters. As shown below this vector of parameters can be estimated by the method of maximum likelihood. Defining $\mathbf{a}_k = \int_0^T \mathbf{q}(\tau) \cos \omega_k \tau \, d\tau$, $\mathbf{b}_k = \int_0^T \mathbf{q}(\tau) \sin \omega_k \tau \, d\tau$, $\mathbf{a}(\omega) = \int_0^T \mathbf{q}(\tau) \cos \omega\tau \, d\tau$, $\mathbf{b}(\omega) = \int_0^T \mathbf{q}(\tau) \sin \omega\tau \, d\tau$, and $\omega_{\max} = N\Delta\omega$, the likelihood function becomes

$$
\begin{aligned}
L(\boldsymbol{\mu}|\mathbf{q}) &\propto \prod_{k=1}^{N} \frac{1}{\det [\mathbf{S}_k]} \exp\left[-\frac{2}{T^2\Delta\omega}(\mathbf{a}_k{}^\mathsf{T}\mathbf{S}^{-1}(\omega_k; \boldsymbol{\mu})\,\mathbf{a}_k + \mathbf{b}_k{}^\mathsf{T}\mathbf{S}^{-1}(\omega_k; \boldsymbol{\mu})\,\mathbf{b}_k)\right] \\
&\approx \exp\left[-\frac{T}{2\pi} \int_0^{\omega_{\max}} \log[\det \mathbf{S}(\omega; \boldsymbol{\mu})] \, d\omega\right. \\
&\quad \left. -\frac{1}{2\pi^2} \int_0^{\omega_{\max}} [\mathbf{a}(\omega)^\mathsf{T}\mathbf{S}^{-1}(\omega; \boldsymbol{\mu})\,\mathbf{a}(\omega) + \mathbf{b}(\omega)^\mathsf{T}\mathbf{S}^{-1}(\omega; \boldsymbol{\mu})\,\mathbf{b}(\omega)] \, d\omega\right]
\end{aligned}
\tag{12}
$$

for a sufficiently small increment $\Delta\omega = 2\pi/T$. According to (Brillinger 1974) use of this likelihood function for spectral parameters dates back to (Whittle 1953). The function

seems not to have been formulated in Maritime Engineering until its independent appearance in (Friis-Hansen & Ditlevsen 2003). For the ship study considered herein (8) shows that $\mathbf{S}(\omega; \boldsymbol{\mu}) = \{\mathfrak{Re}[U_j(\omega, k; \boldsymbol{\mu})U_k^*(\omega, k; \boldsymbol{\mu})] S_e(\omega; \boldsymbol{\mu})\}_{j,k=0,1}, \omega = \omega_e$, because the real part and the imaginary part of $u(t) = u_0(t) + u_1(t)(x - x_g)$ have identical correlation functions.
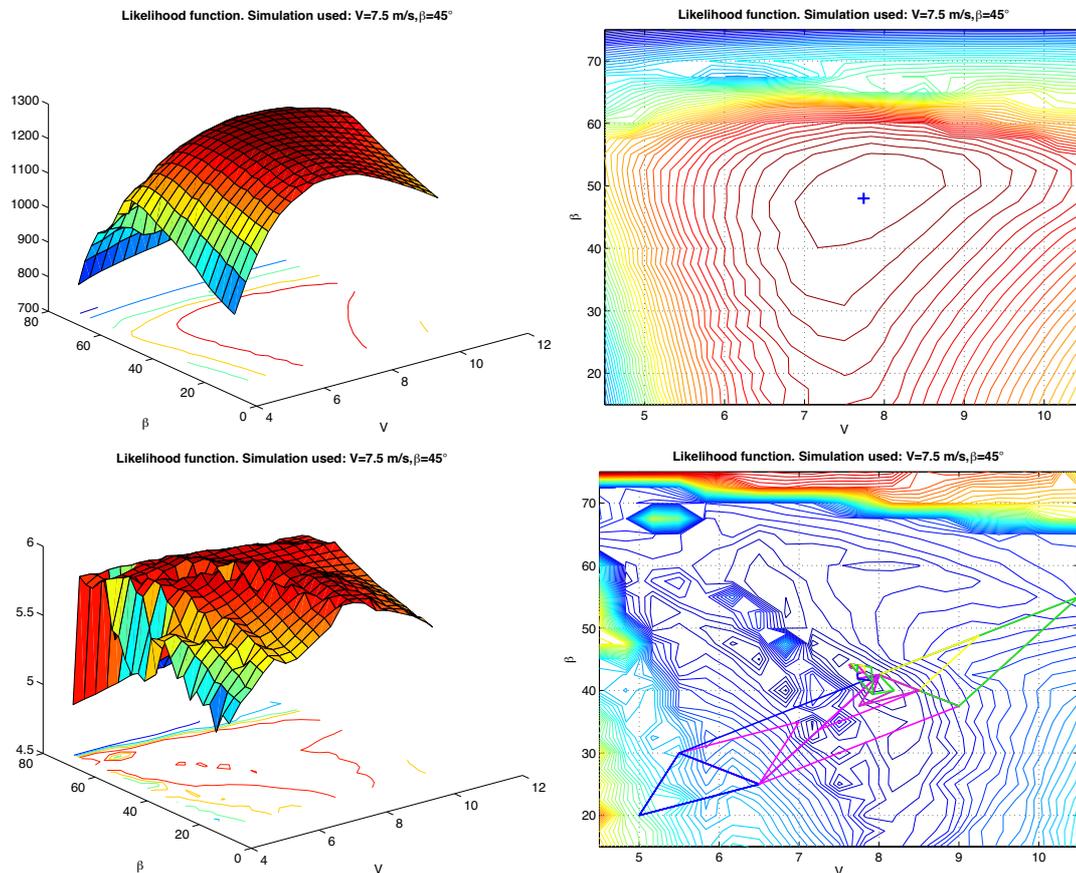

Figure 1: The likelihood (top: pram, bottom: ship) as functions of vessel speed V and heading β, obtained from 10 minutes simulated heave and pitch motions for $\beta = 45°$ towards head sea, $V = 7.5$ m/s, $H_S = 4$ m and $T_Z = 7$ s. The point of maximum likelihood is $(\beta, V) = (48.0°, 7.74$ m/s$)$ for the pram and $(43.8°, 7.65$ m/s$)$ for the ship. The irregular variation of the likelihood functions (in particular so for the ship) makes a gradient based optimization unstable. Here a simple climbing triangular amoeba optimization is used as indicated in the bottom right diagram. The vertices are on the likelihood surface. The triangle can shrink or expand to half or double area by moving any vertex along the corresponding median keeping the base fixed, or by parallel moving any base along the median keeping the vertex and the top angle fixed. Moreover the triangle can flip over with respect to any of its sides. At each step the most advantageous operation is made.

Example and Conclusions

A simple 80 m long and 10 m wide pram-shaped ship structure with 5 m draft is considered, and also the ship "Nakskov", which is an early generation container vessel. The main particulars of the ship are: Length between perpendiculars = 185.93 m, breadth moulded =

25.91 m, depth moulded = 19.50 m, design draught = 8.5 m, difference in draft fore - aft = 1.6 m, block coefficient = 0.63.

For both vessels the heave and pitch motions are simulated as they could occur in a Gaussian sea state with the parameter values $(H_s, T_z) = (4 \text{ m}, 7 \text{ s})$ using the theory described above up to and including (9). The spectral range is defined either in the wave frequency range of ω or in the encounter frequency range of ω_e. The range is selected such that none of the two frequencies exceed the range $[0, \omega_{max}] = [0, 4] \text{ s}^{-1}$. For $\omega_0 < 0$ it is the direct wave frequency that defines the range, whereas the encounter frequency defines the range when $\omega_0 > 0$. For the largest range of the two the spectrum is discretized into N intervals by a step of $\Delta\omega = 0.01 \text{ s}^{-1}$. Thus the period becomes $T = 2/\Delta\omega = 200\,\pi$. For the smallest frequency range the discretization step $\Delta\omega$ is chosen such that also this range is divided into N intervals. Motions of duration equal to T are simulated for the vessel sailing in quartering sea ($45°$ to head sea) with a speed of 7.5 m/s. The added mass is calculated by use of a so-called Lewis transformation that transforms a unit half circle into a ship-like section. This transformation is useful because it allows an analytical estimation of $M(x, \infty)$. The applied damping coefficient $N(x, \omega)$ is derived by Tasai, see (Jensen 2001) for details.

The goal herein is to apply likelihood maximization to estimate the speed V, the heading angle β, and the two sea state parameters from the measured records. To avoid division by zero in the numerical calculations of the likelihood function (12), simulated mutually independent white noises of small common spectral intensity are added to the each of the heave and pitch realizations. The response spectra of the modified motions hereby become lifted above zero by the known white noise intensity, but are otherwise unchanged.

Figure 1 shows the surface and the contour plots of the likelihood functions as obtained from (12) in the case where the sea state parameters are known, that is, the values used for the simulations are substituted into the likelihood function. The irregular fluctuations of the likelihood functions will decrease by decreasing step $\Delta\omega$ at the cost of computer time. The fluctuations make a gradient based optimization unstable. Instead a simple climbing amoeba optimization is applied as indicated in Figure 1 bottom right. With all parameters unknown the likelihood optimization for the simulated motions gave $(\beta, V, H_s, T_z) = (48.6°, 8.10 \text{ m/s}, 3.98 \text{ m}, 7.25 \text{ s})$ for the pram and $(46.5°, 7.28 \text{ m/s}, 4.55 \text{ m}, 6.88 \text{ s})$ for the ship. The conclusion is that the obtained estimates are reasonably accurate giving promises about useful applications of fast information extracting from monitored response processes.

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